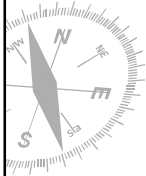


Math 119 – Plane Geometry

Section 1.7 Formal Proof

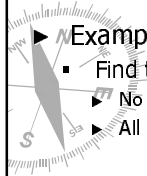


Reminder: Conditional Statements

- ▶ If H, then C.
 - Short hand: $H \Rightarrow C$
 - H = Hypothesis = what you are given
 - C = Conclusion = what you need to prove

▶ Example

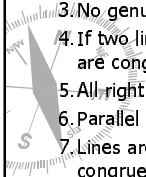
- Find the conclusion/hypothesis for:
 - ▶ No ghost has a shadow.
 - ▶ All straight angles are congruent.



Exercise:

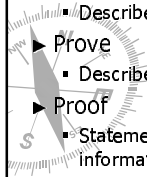
- ▶ Find the hypothesis and conclusion for each statement (Hint: You may need to rephrase them):
 1. All leap years have 366 days.
 2. Use the stairs instead of the elevator in case of fire.
 3. No genuine phone number begins with 555.
 4. If two lines intersect, then the vertical angles formed are congruent.
 5. All right angles are congruent.
 6. Parallel lines do not intersect.
 7. Lines are perpendicular when they meet to form congruent adjacent angles.

Try 1.7 #1-8



Parts of a Proof

- ▶ Statement of the Theorem
- ▶ Drawing
 - Represents the Hypothesis of the Theorem
- ▶ Given
 - Describes Hypothesis in terms of Drawing
- ▶ Prove
 - Describes Conclusion in terms of Drawing
- ▶ Proof
 - Statements justified by reasons that begin with Given information and end with Prove statement



Practice: Given/Drawing/Prove

- ▶ For each theorem, make a Drawing. On the basis of your Drawing, write a Given and a Prove for the theorem.
- 1. If two angles are complementary to the same angle, then these angles are congruent.
- 2. If two lines intersect, then the vertical angles are congruent.
- 3. All right angles are congruent.
- 4. Parallel lines do not intersect.
- 5. Lines are perpendicular when they meet to form congruent adjacent angles.

Try 1.7 #9-14

Unwritten Part of Proof: The PLAN Getting from Prove to Proof

- ▶ To develop a PLAN, it is:
 - 1) Helpful to look at conclusions (possible later statements) to which the Given information would lead
 - I.e: What does the Given give you?
 - 2) Helpful to reverse the order by asking the question, "What was necessary (possible earlier statements) for the Prove to be true?"
 - I.e: What will get you to the Prove?
 - Ask: The Prove statement would be true if what else were true?

Items You May Use as Reasons in Proofs/Plans

- ▶ Previously defined words
- ▶ Previously stated postulates
- ▶ Previously proved theorems
 - NEVER state them by their number. Always write the statement out.
- ▶ Properties from Algebra
- ▶ **DO NOT USE:**
 - Theorems we have not yet proven
- ▶ **Final Note:** There are many right answers

Theorem 1.7.2: If two lines meet to form a right angle, then these lines are perpendicular.

- ▶ Make a Drawing
- ▶ Pick out the Given
- ▶ Determine the Prove
- ▶ Form a PLAN
 - What can we deduce from our Given?
 - What do we need in order to show the lines are perpendicular?
 - ▶ I.e: What does it mean to be perpendicular?
 - How can we put these together?

Practice: PLAN

- ▶ Using the Given/Prove/Drawing you came up with, try to come up with a PLAN for each of the following:
 1. If two angles are complementary to the same angle, then these angles are congruent.
 2. If two lines intersect, then the vertical angles are congruent.
 3. All right angles are congruent.
 4. Parallel lines do not intersect.
 5. Lines are perpendicular when they meet to form congruent adjacent angles.

Try 1.7 #21-29

Arrangement of Final Proof

- ▶ Statements must follow from earlier statements deductively
 - Statement in Proof (Hypothesis)
 - Reason for Statement (Principle)
 -
 - ∴ Next Statement in Proof (Conclusion)

Format of Proof of a Theorem

- ▶ Theorem: If P, then Q.
- ▶ Drawing (Use hypothesis P)
- ▶ Given: Specific names on Drawing describe hypothesis
- ▶ Prove: Specific names on Drawing describe conclusion

PROOF	
Statements	Reasons

Formal Proof for Theorem 1.7.2

Theorems from 1.7

- ▶ **Thm 1.7.1:** If two lines are perpendicular, then they meet to form right angles.
- ▶ **Thm 1.7.2:** If two lines meet to form a right angle, then these lines are perpendicular.
- ▶ **Thm 1.7.3:** If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.
- ▶ **Thm 1.7.4:** If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.
- ▶ **Thm 1.7.5:** If two lines intersect, then the vertical angles formed are congruent.

Theorems from 1.7

- ▶ **Thm 1.7.6:** Any two right angles are congruent.
- ▶ **Thm 1.7.7:** If the exterior sides of two adjacent acute angles form perpendicular rays, then these angles are complementary.
- ▶ **Thm 1.7.8:** If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.
- ▶ **Thm 1.7.9:** If two segments are congruent, then their midpoints separate these segments into four congruent segments.
- ▶ **Thm 1.7.10:** If two angles are congruent, then their bisectors separate these angles into four congruent angles.

Try 1.7 #15-20

Homework

- ▶ Due Monday 6/21
 - Read Section 1.7
 - 1.7: # 1-29

▶ REMINDER: Monday we meet in AA-145!