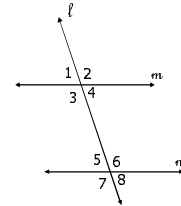


# Math 119 – Plane Geometry

Sections 2.3, 2.4 and 2.5  
Parallel Lines 2  
6/22/2004

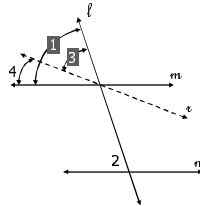
## Example: Warm-up

- ▶ Name all angles congruent to  $\angle 1$ .
- ▶ Name all angles congruent to  $\angle 6$ .
- ▶ Write an equation relating  $\angle 3$  and  $\angle 5$ .



**Theorem 2.3.1:** If two lines are cut by a transversal so that the corresponding angles are congruent, then these lines are parallel.

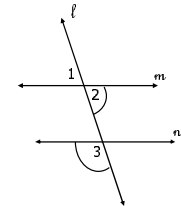
- ▶ Given:  $n$  and  $m$  with transversal  $\ell$   
 $\angle 1 \cong \angle 2$
- ▶ Prove:  $n \parallel m$



Use INDIRECT Proof

**Theorem 2.3.2:** If two lines are cut by a transversal so that the alternate interior angles are congruent, then these lines are parallel.

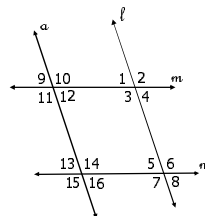
- ▶ Given:  $n$  and  $m$  with transversal  $\ell$   
 $\angle 2 \cong \angle 3$
- ▶ Prove:  $n \parallel m$



- ▶ **Theorem 2.3.3:** If two lines are cut by a transversal so that the alternate exterior angles are congruent, then these lines are parallel. (HW)

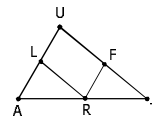
## Example

- ▶ Which lines must be parallel if  $\angle 4 \cong \angle 5$ ?
- ▶ If  $\angle 9 \cong \angle 16$ , must  $\angle 1$  and  $\angle 10$  be supplementary?
- ▶ If  $\angle 6 \cong \angle 15$ , must  $\angle 1$  and  $\angle 10$  be supplementary?



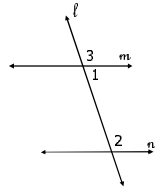
## Example

- ▶ Which line segments are parallel if...
  1. ...  $m\angle LRF = m\angle RFT$ ?
  2. ...  $\angle U$  and  $\angle UFR$  are supplementary?
  3. ...  $m\angle A = m\angle FRT$ ?



**Theorem 2.3.4:** If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then these lines are parallel.

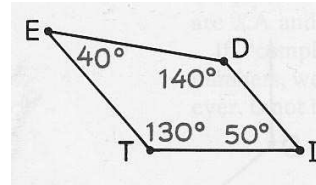
- Given:  $n$  and  $m$  with transversal  $\ell$   
 $\angle 1$  is supplementary to  $\angle 2$



- Prove:  $n \parallel m$
- **Theorem 2.3.5:** If two lines are cut by a transversal so that the exterior angles on the same side of the transversal are supplementary, then these lines are parallel. (HW)

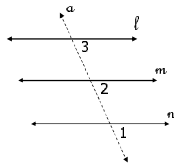
### Example

- Which lines must be parallel?



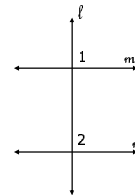
**Theorem 2.3.6:** If two lines are each parallel to a third line, then these lines are parallel to each other.

- Given:  $\ell \parallel m$  and  $n \parallel m$  with transversal  $a$
- Prove:  $\ell \parallel n$



**Theorem 2.3.7:** If two coplanar lines are each perpendicular to a third line, then these lines are parallel to each other.

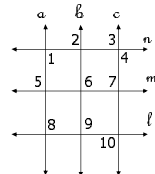
- Given:  $m \perp \ell$  and  $n \perp \ell$
- Prove:  $m \parallel n$



### Example

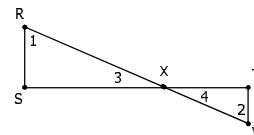
- If possible, name a pair of lines that must be parallel if each of the following is true:

- $m\angle 1 = m\angle 3$
- $m\angle 2 = m\angle 5$
- $m\angle 3 = m\angle 7$
- $\ell \perp \ell$  and  $\ell \perp c$
- $m\angle 5 = m\angle 6$
- $\angle 1$  and  $\angle 8$  are supplementary
- $m\angle 3 = m\angle 4$
- $\angle 5$  and  $\angle 7$  are supplementary
- $m\angle 8 = m\angle 10$
- $a \perp n$  and  $b \perp m$



### Example: A Proof

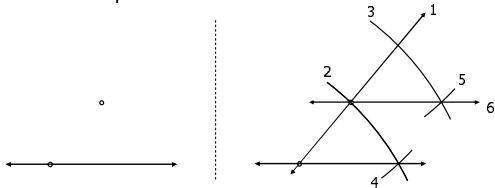
- Given:  $RV$  and  $ST$  intersect at point  $X$   
 $\angle 1$  and  $\angle 3$  are complementary  
 $\angle 2$  and  $\angle 4$  are complementary
- Prove:  $RS \parallel TV$



## Construction: Parallel Lines

► Use the following theorem:

- If two lines are cut by a transversal so that corresponding angles are congruent, then these lines are parallel.

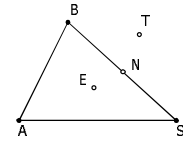


## Triangles

► **Def:** A **triangle** is the union of three line segments that are determined by three noncollinear points.

- The points are the **vertices** of the triangle.
- The line segments are the **sides** of the triangle.

1. Name a point *on* the triangle, *in the interior* of the triangle, and *in the exterior* of the triangle.



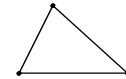
## Example

- How do we know the union of line segments BA, BT and AT does not form a triangle?

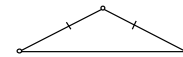


## Triangles Classified By Sides

- A **scalene triangle** has no congruent sides.



- An **isosceles triangle** has two congruent sides.

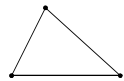


- An **equilateral triangle** has three congruent sides.

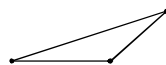


## Triangles Classified By Angles

- An **acute triangle** has three acute angles.



- An **obtuse triangle** has one obtuse angle.



- An **equiangular triangle** has three congruent angles.

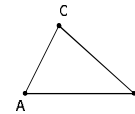


- A **right triangle** contains one right angle.

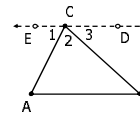


**Theorem 2.4.1:** In a triangle, the sum of the measures of the interior angles is 180.

- Given:  $\triangle ABC$



- Prove:  $m\angle A + m\angle B = m\angle C$

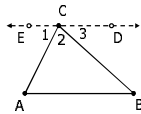


## Auxiliary/Helping Lines

- ▶ When constructed, a helping line enables one to solve a problem or complete a proof.
- ▶  $\overline{ED}$  from the last proof was a helping line

### Justify via theorems:

- "Through a point outside a line, there is exactly one line parallel to the given line."
- Shorthand – "By construction."



## Corollaries To Sum of Angles

- ▶ **Cor 2.4.2:** Each angle of an equiangular triangle measures 60. (HW)
- ▶ **Cor 2.4.3:** The acute angles of a right triangle are complementary. (HW)
- ▶ **Cor 2.4.4:** If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.
  - Discuss why true.

## Example

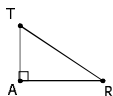
- ▶ In  $\triangle TEA$ ,  $m\angle T = 10$  and  $m\angle E = 20$ .
  - Find  $m\angle A$ .
  - What kind of triangle is  $\triangle TEA$  with respect to its angles?
- ▶ In  $\triangle WEB$ ,  $m\angle W = 40$  and  $m\angle E = 70$ .
  - Find  $m\angle B$ .
  - What kind of triangle is  $\triangle WEB$  with respect to its angles?

## Example

- ▶ In  $\triangle SAD$ ,  $\angle S \cong \angle A \cong \angle D$ .
  - What kind of triangle is  $\triangle SAD$  with respect to its angles?
  - What is the measure of each angle of  $\triangle SAD$ ?
- ▶ In  $\triangle WED$  and  $\triangle BOX$ ,  $\angle W \cong \angle B$  and  $\angle E \cong \angle O$ .
  - What can you conclude about  $\angle D$  and  $\angle X$ ?
- ▶ In  $\triangle LIE$ ,  $\angle L$  is a right angle.
  - What relation do  $\angle I$  and  $\angle E$  have?

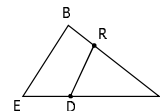
## Example

- ▶  $\overline{TA} \perp \overline{AR}$ .
- ▶  $m\angle R = (3x + 5)$  and  $m\angle T = (4x - 6)$ 
  - Find  $x$ .
  - Find  $m\angle R$  and  $m\angle T$



## Example: Proof

- ▶ Given: In  $\triangle BIE$  and  $\triangle DRI$ ,  $m\angle B = m\angle DRI$
- ▶ Prove:  $m\angle E = m\angle IRD$



### Example: Proof

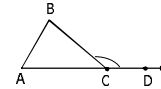
- ▶ Given:  $\overline{NU} \perp \overline{UT}$
- ▶ Prove:  $\angle N + \angle T = 90$



### Exterior Angles

- ▶ When the sides of a triangle are extended, each angle that is formed by a side and an extension of the adjacent side is an **exterior angle** of the triangle.

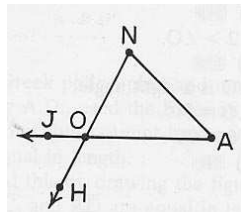
- ▶ How does  $\angle DCB$  relate to  $\angle A$  and  $\angle B$ ? Why?



### Exterior Angles – Example

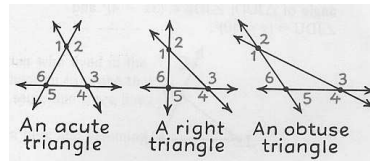
- ▶ In this figure, JON and HOA are exterior angles

  - Name a relation between angles JON and NOA
  - Name a relation between angles JON and HOA
  - Is JOH an exterior angle of the triangle?
  - How many exterior angles does a triangle have at one vertex?
  - How many exterior angles does a triangle have in all?



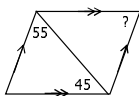
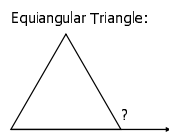
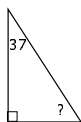
### Example

- ▶ Use the following pictures to decide T/F:



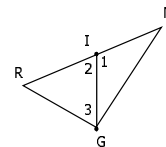
- All six exterior angles of a triangle may be obtuse.
- Some of the exterior angles of an obtuse triangle are acute.
- All six exterior angles of a triangle may have different measures.
- An exterior angle of a triangle may be smaller than one of the nonadjacent interior angles of the triangle.
- An exterior angle of a triangle may be equal to one of the interior angles of the triangle.

### Example: Find the Angle



### Example: Proof

- ▶ Given: R-I-N
- $\angle 1$  and  $\angle 2$  are adjacent
- ▶ Prove:  $m\angle R = m\angle 1 - m\angle 3$



## Homework

- ▶ Due Wednesday 6/23/2004
  - Read Sections 2.3, 2.4 and 2.5
  - 2.3: #1-29, 31-33
  - 2.4: #1-29, 34-40