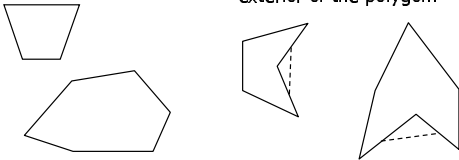


Math 119 – Plane Geometry

Sections 2.5 and 3.1
Polygons and Triangles I
6/23/2004

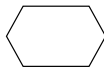
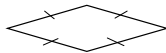
Polygons

- ▶ **Def:** A **polygon** is a closed plane figure whose sides are line segments that intersect only at the endpoints (**vertices**).
- ▶ **Convex polygons** are polygons that have angles that measure between 0 and 180.
- ▶ **Concave polygons** are those polygons in which a line segment joining two of its points can lie in the exterior of the polygon.



Types of Polygons

- ▶ An **equilateral polygon** is one in which all sides are congruent.
- ▶ An **equiangular polygon** is one in which all angles are congruent.
- ▶ A **regular polygon** is one which is both equilateral and equiangular.



Number of Sides of Polygons

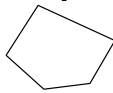
3 sides
Triangle



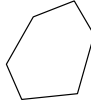
4 sides
Quadrilateral



5 sides
Pentagon



6 sides
Hexagon



7 sides
Heptagon



8 sides
Octagon



9 sides
Nonagon



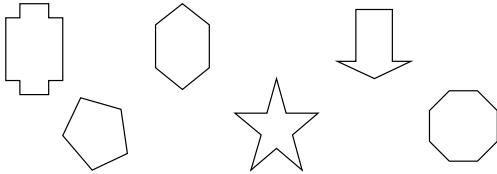
10 sides
Decagon



What shape does the polygon appear to become as the number of sides increases?

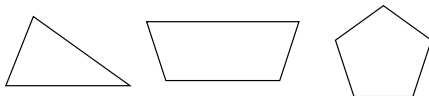
Example

- ▶ Name each polygon according to the number of sides.
- ▶ Which polygons are convex?



Diagonals of a Polygon

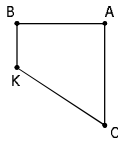
- ▶ **Def:** A **diagonal** of a polygon is a line segment that joins two nonconsecutive vertices.
- ▶ Count the number of diagonals for each polygon:



- ▶ Evaluate $n(n-3)/2$ for each polygon where n is the number of sides.

Example

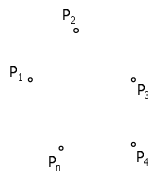
- What are \overline{B} , \overline{A} , \overline{C} , and \overline{K} called with respect to the polygon?
- What are \overline{CK} , \overline{CA} , \overline{AB} , and \overline{BK} called?
- If \overline{BC} and \overline{AK} were drawn, what would they be called?
- What type of polygon is \overline{BACK} : *convex* or *concave*?
- What is \overline{BACK} called with respect to the number of sides?



Theorem 2.5.1: The total number of diagonals for a polygon is given by $D = n(n-3)/2$.

► Outline of Proof:

- Choose one point.
 - How many diagonals can be drawn?
- Repeat for each point.
 - How many times can we choose a new point?
- How many times did we count each diagonal?

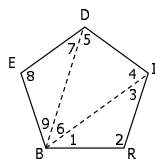


Sum of Interior Angles of a Polygon

► **Theorem 2.5.2:** The sum S of the measures of the interior angles of a polygon with sides n is given by $S = (n - 2) \cdot 180$.

► Outline of Proof:

- Cut polygon into triangles using diagonals from 1 vertex.
- Sum up angles of pentagon
 - Write in terms of the triangles



Example

1. Find the sum of the measures of the interior angles of a pentagon.
2. Find the measure of each interior angle of an equiangular pentagon.
3. Find the number of sides in a polygon whose sum of interior angles is 1800.
4. Is it possible to have a polygon with a sum of interior angles equal to 2000? Why/why not?

Interior Angle of Regular Polygon

- **Cor 2.5.3:** The measure I of each interior angle of a regular polygon of n sides is:

$$I = [(n - 2) \cdot 180] / n$$

- **Ex:** Find the measure of one interior angle of a regular decagon.
- **Ex:** Each interior angle of a certain polygon has measure 150. Find its number of sides.

Corollaries:

- **Cor 2.5.4:** The sum of the four interior angles of a quadrilateral is 360.
- **Cor 2.5.5:** The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.
- To prove, use:

$$\begin{array}{l} \text{Sum of Measures} \\ \text{Of Interior Angles} \end{array} + \begin{array}{l} \text{Sum of Measures} \\ \text{Of Exterior Angles} \end{array} = \begin{array}{l} \text{Sum of Measures of All} \\ \text{Supplementary Pairs} \end{array}$$

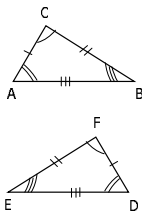
- **Cor 2.5.6:** The measure E of each exterior angle of a regular polygon of n sides is $E = 360/n$.

Examples

- ▶ Find the sum of the exterior angles of a regular pentagon.
- ▶ Use Cor 2.5.5 to find the number of sides of a regular polygon if each interior angle measures 150.
 - Yes, we are repeating a previous example.

Congruent Triangles

- ▶ Two triangles are **congruent** if one coincides with (fits perfectly over) the other.



▶ $\triangle ABC \cong \triangle DEF$ means...

- $\angle A \cong \angle D$
- $\angle B \cong \angle E$
- $\angle C \cong \angle F$
- $\overline{AB} \cong \overline{DE}$
- $\overline{BC} \cong \overline{EF}$
- $\overline{AC} \cong \overline{DF}$

ORDER MATTERS!!!!!!!

Write: $A \leftrightarrow D$ $B \leftrightarrow E$ $C \leftrightarrow F$

Definition of Congruent Triangles

- ▶ **Def:** Two triangles are **congruent** when the six parts of the first triangle are congruent to the six corresponding parts of the second triangle.
- ▶ **Ex:** If $R \leftrightarrow L$ $S \leftrightarrow M$ $T \leftrightarrow N$, what may we conclude?
- ▶ **Important!** If the triangles are congruent, the corresponding parts are congruent also.

Properties of Congruent Triangles

► Reflexive

- Any triangle is congruent to itself
- $\triangle ABC \cong \triangle ABC$

Q: What kind of a relation is Congruence of Triangles?

► Symmetric

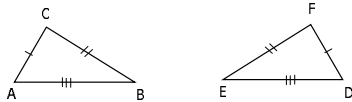
- If a triangle is congruent to a 2nd triangle, then the 2nd triangle is congruent to the 1st.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

► Transitive

- If one triangle is congruent to a 2nd triangle, and the 2nd to a 3rd, then the 1st is congruent to the 3rd.
- If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHK$, then $\triangle ABC \cong \triangle GHK$.

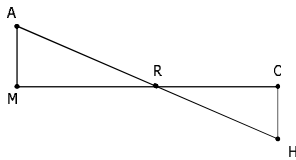
Method for Proving Congruence: SSS

- **Post. 12:** If the three sides of one triangle are congruent to the three sides of a second triangle, the triangles are congruent (SSS).



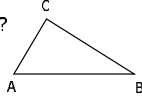
Example

- Given: \overline{AB} and \overline{CD} bisect each other at M
 $\overline{AC} \cong \overline{DB}$
- Prove: $\triangle AMC \cong \triangle BMD$



Included Sides/Angles

- ▶ The two sides that form an angle of a triangle are said to **include that angle** of the triangle.
- ▶ Two angles of a triangle are said to **include** their common side.
- ▶ Which angle is included by AC and CB?
- ▶ Which sides include $\angle B$?
- ▶ What is the included side for $\angle A$ and $\angle B$?
- ▶ Which angles include CB?



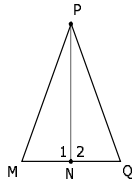
Method for Proving Congruence: SAS

- ▶ **Post 13:** If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent. (SAS)



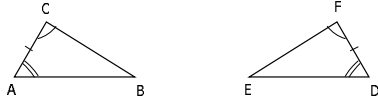
Example

- ▶ Given: $\overline{PN} \perp \overline{MQ}$
 $\overline{MN} \cong \overline{NQ}$
- ▶ Prove: $\triangle PNM \cong \triangle PNQ$



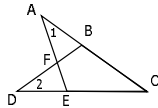
Method for Proving Congruence: ASA

► **Post 14:** If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent. (ASA)



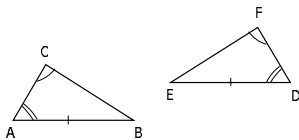
Example

- Given: $AC \cong DC$
 $\angle 1 \cong \angle 2$
- Prove: $\triangle ACE \cong \triangle DCB$



Method for Proving Congruence: AAS

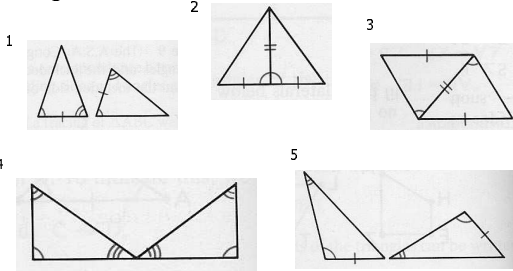
► **Thm 3.1.1:** If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of a second triangle, then the triangles are congruent. (AAS)



Prove this!

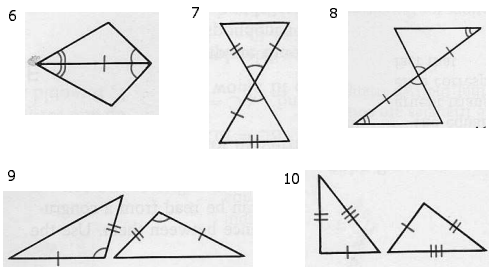
Example

► Is it possible to conclude the triangles are congruent?



Example

► Is it possible to conclude the triangles are congruent?



Homework

► Due Thursday 6/24

- Study for Exam 2 – Sections 1.7, 3.1; Chapter 2
- Suggested Preparation:
 - Read Sections 2.5 and 3.1
 - 2.5: #1-19, 21-24, 29, 34
 - 3.1: #1-38
 - Chapter 1 Review: #30-43
 - Chapter 2 Review: #1-42
 - Review Vocabulary/Theorems/Postulates
