Math 119 – Plane Geometry

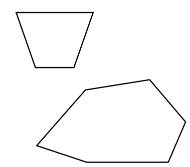
Sections 2.5 and 3.1

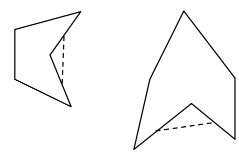
Polygons and Triangles I

6/23/2004

Polygons

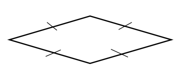
- **Def**: A **polygon** is a closed plane figure whose sides are line segments that intersect only at the endpoints (**vertices**).
  - **Convex polygons** are polygons that have angles that measure between 0 and 180.
  - **Concave polygons** are those polygons in which a line segment joining two of its points can lie in the exterior of the polygon.



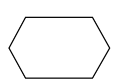


Types of Polygons

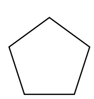
- An **equilateral polygon** is one in which all sides are congruent.



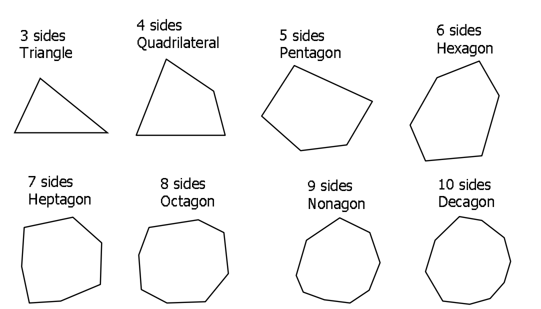
- An **equiangular polygon** is one in which all angles are congruent.



- A **regular polygon** is one which is both equilateral and equiangular.



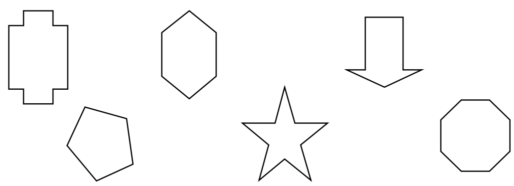
Number of Sides of Polygons



What shape does the polygon appear to become as the number of sides increases?

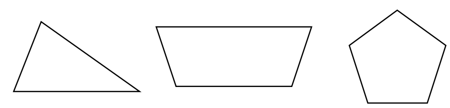
Example

- Name each polygon according to the number of sides.
- Which polygons are convex?



Diagonals of a Polygon

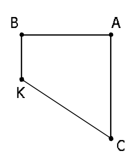
- **<u>Def:</u>** A **<u>diagonal</u>** of a polygon is a line segment that joins two nonconsecutive vertices.
- Count the number of diagonals for each polygon:



- Evaluate n(n-3)/2 for each polygon where n is the number of sides.

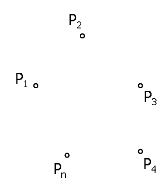
Example

a. What are B, A, C, and K called with respect to the polygon?
b. What are $\overline{CK}$, $\overline{CA}$, $\overline{AB}$, and $\overline{BK}$ called?
c. If $\overline{BC}$ and $\overline{AK}$ were drawn, what would they be called?
d. What type of polygon is BACK: *convex* or *concave?*
e. What is BACK called with respect to the number of sides?



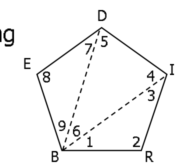
<u>Theorem 2.5.1</u>: The total number of diagonals for a polygon is given by D = n(n-3)/2.

- Outline of Proof:
  - Choose one point.
    - How many diagonals can be drawn?
  - Repeat for each point.
    - How many times can we choose a new point?
  - How many times did we count each diagonal?



Sum of Interior Angles of a Polygon

- **Theorem 2.5.2**: The sum S of the measures of the interior angles of a polygon with sides n is given by S = (n – 2)·180.
- Outline of Proof:
  - Cut polygon into triangles using diagonals from 1 vertex.
  - Sum up angles of pentagon
    - Write in terms of the triangles



Example

1. Find the sum of the measures of the interior angles of a pentagon.
2. Find the measure of each interior angle of an equiangular pentagon.
3. Find the number of sides in a polygon whose sum of interior angles is 1800.
4. Is it possible to have a polygon with a sum of interior angles equal to 2000? Why/why not?

Interior Angle of Regular Polygon

- **Cor 2.5.3**: The measure I of each interior angle of a regular polygon of n sides is:

  I = [(n – 2)·180]/n
- **Ex**: Find the measure of one interior angle of a regular decagon.
- **Ex**: Each interior angle of a certain polygon has measure 150. Find its number of sides.

Corollaries:

- **Cor 2.5.4**: The sum of the four interior angles of a quadrilateral is 360.
- **Cor 2.5.5**: The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.
  - To prove, use:

Sum of Measures Of Interior Angles + Sum of Measures Of Exterior Angles = Sum of Measures of All Supplementary Pairs

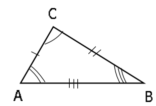
- **Cor 2.5.6**: The measure E of each exterior angle of a regular polygon of n sides is E = 360/n.

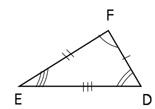
Examples

- Find the sum of the exterior angles of a regular pentagon.
- Use Cor 2.5.5 to find the number of sides of a regular polygon if each interior angle measures 150.
  - Yes, we are repeating a previous example.

Congruent Triangles

- Two triangles are **congruent** if one coincides with (fits perfectly over) the other.





- $\Delta ABC \cong \Delta DEF$ means…
  - $\angle A \cong \angle D$
  - $\angle B \cong \angle E$
  - $\angle C \cong \angle F$
  - $\overline{AB} \cong \overline{DE}$
  - $\overline{BC} \cong \overline{EF}$
  - $\overline{AC} \cong \overline{DF}$

ORDER MATTERS!!!!!!!!!

Write: $A \leftrightarrow D$ $B \leftrightarrow E$ $C \leftrightarrow F$

Definition of Congruent Triangles

- **Def**: Two triangles are **congruent** when the six parts of the first triangle are congruent to the six corresponding parts of the second triangle.
- **Ex**: If $R \leftrightarrow L$ $S \leftrightarrow M$ $T \leftrightarrow N$, what may we conclude?
- Important! If the triangles are congruent, the corresponding parts are congruent also.

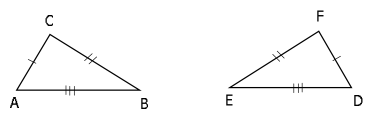
Properties of Congruent Triangles

Q: What kind of a relation is Congruence of Triangles?

- Reflexive
  - Any triangle is congruent to itself
  - $\Delta ABC \cong \Delta ABC$
- Symmetric
  - If a triangle is congruent to a 2^{nd} triangle, then the 2^{nd} triangle is congruent to the 1^{st}.
  - If $\Delta ABC \cong \Delta DEF$, then $\Delta DEF \cong \Delta ABC$.
- Transitive
  - If one triangle is congruent to a 2^{nd} triangle, and the 2^{nd} to a 3^{rd}, then the 1^{st} is congruent to the 3^{rd}.
  - If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta GHK$, then $\Delta ABC \cong \Delta GHK$.

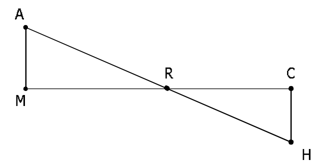
Method for Proving Congruence: SSS

- **Post. 12**: If the three sides of one triangle are congruent to the three sides of a second triangle, the triangles are congruent (SSS).



Example

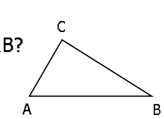
- Given: $\overline{AB}$ and $\overline{CD}$ bisect each other at M
  $\overline{AC} \cong \overline{DB}$
- Prove: $\Delta AMC \cong \Delta BMD$



Included Sides/Angles

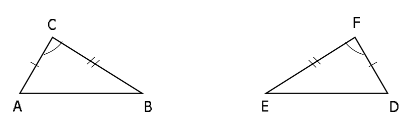
- The two sides that form an angle of a triangle are said to **include that angle** of the triangle.
- Two angles of a triangle are said to **include** their common side.

- Which angle is included by AC and CB?
- Which sides include ∠B?
- What is the included side for ∠A and ∠B?
- Which angles include CB?



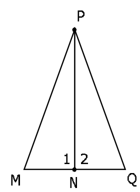
Method for Proving Congruence: SAS

- **Post 13**: If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent. (SAS)



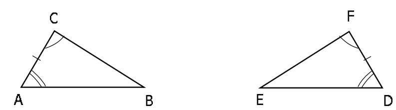
Example

- Given: $\overline{PN} \perp \overline{MQ}$
  $\overline{MN} \cong \overline{NQ}$
- Prove: $\Delta PNM \cong \Delta PNQ$



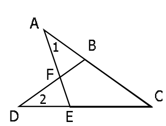
Method for Proving Congruence: ASA

- **<u>Post 14</u>**: If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent. (ASA)



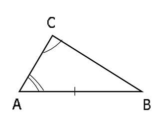
Example

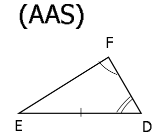
- Given: $AC \cong DC$
  $\angle 1 \cong \angle 2$
- Prove: $\Delta ACE \cong \Delta DCB$



Method for Proving Congruence: AAS

- **<u>Thm 3.1.1</u>**: If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of a second triangle, then the triangles are congruent. (AAS)





Prove this!

Example

- Is it possible to conclude the triangles are congruent?

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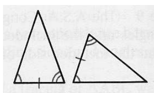

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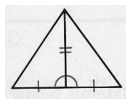

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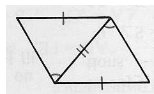

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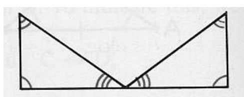

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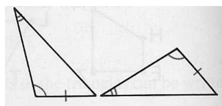

Example

- Is it possible to conclude the triangles are congruent?

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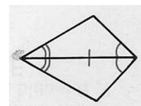

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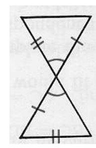

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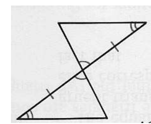

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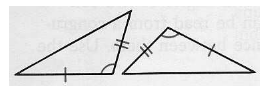

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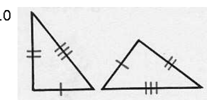

Homework

- Due Thursday 6/24
  - Study for Exam 2 – Sections 1.7, 3.1; Chapter 2
  - Suggested Preparation:
    - Read Sections 2.5 and 3.1
    - 2.5: #1-19, 21-24, 29, 34
    - 3.1: #1-38
    - Chapter 1 Review: #30-43
    - Chapter 2 Review: #1-42
    - Review Vocabulary/Theorems/Postulates