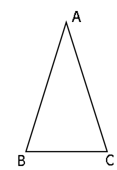
Math 119 – Plane Geometry

Sections 3.3, 3.4, and 3.5
Triangles III
6/28/2004

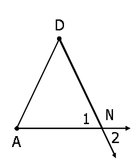
Perimeter

- The **perimeter** of a triangle is the sum of the lengths of the sides.
- Example
  - Given: ∠B ≅ ∠C
    AB = 5.3
    BC = 3.6
  - Find: The perimeter of Δ ABC



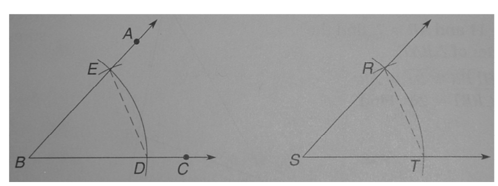
Warm Up Example

- Given: DA = DN
  ∠1 and ∠2 are vertical angles
- Prove: ∠A ≅ ∠2



Justification: Construction of Congruent Angles

- Given: ∠ABC
  $\overline{BD} \cong \overline{BE} \cong \overline{ST} \cong \overline{SR}$
  $\overline{DE} \cong \overline{TR}$
- Prove: ∠B ≅ ∠S

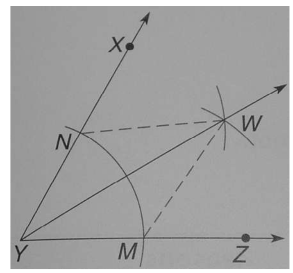


Example

- Construct an isosceles triangle in which obtuse ∠A is included by two sides of length a.

Justification: Construction of Angle Bisector

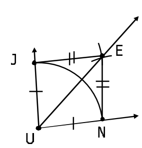
- Given: ∠XYZ
  $\overline{YM} \cong \overline{YN}$
  $\overline{MW} \cong \overline{NW}$
- Prove:
  $\overrightarrow{YW}$ bisects ∠XYZ



Example:

- JU = UN and JE = EN

1. Which two points are equidistant from the other two points?
2. Why is ΔJUE ≅ ΔNUE?
3. Why is ∠JUE ≅ ∠NUE?
4. Why does UE bisect ∠JUN?

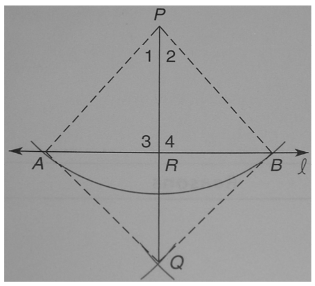


Example: Constructions

- Construct an angle measuring 30.

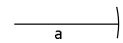
Justification: Construction of Perpendicular Lines

- Given: P not on ℓ
  - $\overline{PA} \cong \overline{PB}$
  - $\overline{AQ} \cong \overline{BQ}$
- Prove: $\overline{PQ} \perp \overline{AB}$



Example: Construction of a Regular Polygon

- Construct a regular hexagon having sides of length a:
  - All sides must be congruent
  - Interior Angles = [(n-2)·180]/n
  - Exterior Angles = 360/n



Example

- Construct an angle measuring 15°.

Inequalities

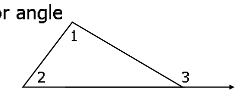
- **Def**: Let a and b be real numbers. a>b if and only if there is a positive number p for which a = b + p.
- **Ex**: Is 2 < 3? Why?
- **Ex**: Is 4 < 2? Why?

Algebraic Properties of Inequality

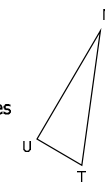
- **Addition**:
  - If a > b and c > d, then a + c > b + d.
- **Subtraction**:
  - If a < b and c = d, then c – a > d – b.
- **Multiplication**:
  - If a < b and c is positive, then ac < bc.
  - If a < b and c is negative, then ac > bc.

More Lemmas

- **Lemma 3.5.3**: If ∠3 is an exterior angle of a triangle and ∠1 and ∠2 are the nonadjacent interior angles, then m∠3 > m∠1 and m∠3 > m∠2.
  - Exterior angle > either exterior angle
  - **Exterior Angle Theorem**

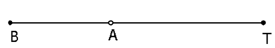


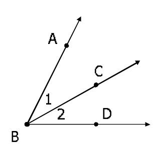
- **Lemma 3.5.4**: In ΔNUT, if ∠U is a right angle or an obtuse angle, then m∠U > m∠T and m∠U > m∠N.
  - Right/obtuse angle > other interior angles



Lemmas – Helping Theorems

- The whole is greater than a part
  - **Lemma 3.5.1**: If A is between B and T on $\overline{BT}$, then BT > BA and BT > AT.
  - **Lemma 3.5.2**: If $\overrightarrow{BD}$ separates ∠ABD into two parts (∠1 and ∠2), then m∠ABD > m∠1 and m∠ABD < m∠2.

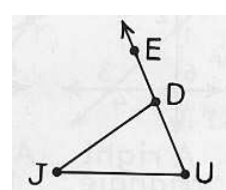




Example

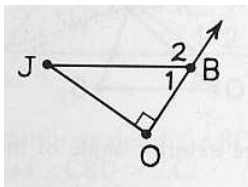
- JDE is an exterior angle of triangle JUD
- Angle JDE = (3x – 4); Angle JDU = (x + 40)

a. Write the inequality that follows from the Exterior Angle Theorem.
b. Find x.
c. What can you conclude about U?
d. Find JDU.



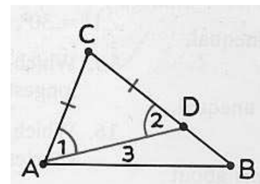
Example – A Proof

- **Given**: ∠2 is an exterior angle
  $\overline{JO}$ is perpendicular to $\overline{OB}$
- **Prove**: ∠2 is obtuse



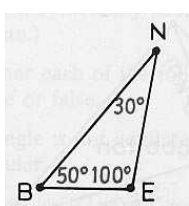
Thm 3.5.6: If one side of a triangle is longer than a second side, then the measure of the angle opposite the longer is greater than the measure of the angle opposite the shorter side.

- **Given**:
  BC > AC
- **Prove**:
  m∠A > m∠B
- **Construct**:
  Segment $\overline{CD}$ on $\overline{CB}$ so that CD = CA



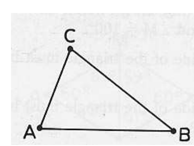
Example

a. Which angle of triangle BEN is largest?
b. Which side of triangle BEN appears longest?
c. What is the relation of the largest angle of triangle BEN to the longest side?
d. Which side of BEN appears the shortest?
e. Which angle of BEN is the smallest?
f. What is the relation of the shortest angle of BEN to the smallest angle?



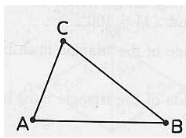
Trichotomy and Converse of 3.5.7

- **The Trichotomy Principle**: If a and b are two numbers, then exactly one a < b, a = b, or a > b is true.
- **Theorem 3.5.7** (The Converse of 3.5.6): If the measure of one angle of a triangle is greater than the measure of a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.
  - **Given**: m∠A > m∠B
  - **Prove**: BC > AC
  - **Plan**: Indirect Proof
  - Eliminate other options of Trichotomy Principle



Given: m∠A > m∠B. Prove: BC > AC

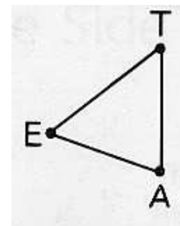
- Assume BC is NOT greater than AC
- Two ways to be NOT greater than:
  1. Assume BC − AC
     - What kind of triangle does this make ABC?
     - What does that mean about angles A and B?
     - Why is this a contradiction?
  2. Assume BC < AC
     - What does the last theorem then tell us about angles A and B?
     - Why is this a contradiction?
- What do these contradictions tell us about the assumption with which we started the proof?
- What conclusion follows?



Example

- In triangle TEA, TA > EA and A > E

a. What can you conclude about angles E and T?
b. What can you conclude about angles A and T?
c. What can you conclude about TE and EA?



Example

- Complete the following statements
  a. In a triangle the measure of an exterior angle ___
  b. If the lengths of one side of a triangle is greater than a second side, ___
  c. If the measure of one angle of a triangle is greater than the measure of a second angle, ___
  d. The Trichotomy Principle: If a and b are two numbers, then either a < b, ___

Example

- Given ΔALE with AL = 4 cm, AE = 5 cm, and LE = 3 cm.

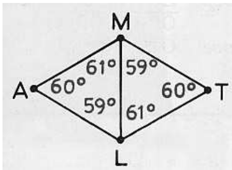
a. Create a rough sketch of the triangle.
b. Which angle of the triangle must be the largest?
c. Which angle of the triangle must be the smallest?

- Given ΔRUM with m∠R = 50°, m∠U = 30°, and m∠M = 100°

a. Which side of the triangle must be the longest?
b. Which side of the triangle must be the shortest?

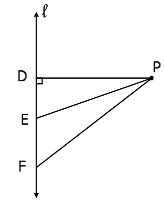
Example

a. Are the triangles congruent?
b. Which side of ΔMAL is the longest?
c. Which side of ΔMLT is the longest?
d. Do these two segments necessarily have equal lengths?



Distance

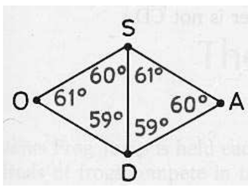
► **Cor 3.5.8**: The perpendicular line segment from a point to a line is the shortest segment that can be drawn from the point to the line.



► If PD⊥ℓ, PD is the **distance** from P to ℓ.

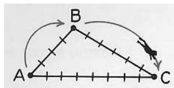
Example

a. Which side of ΔSOD is the longest?
b. Which side of ΔSDA is the longest?
c. Can you draw any conclusions about the relative lengths of these two segments? If so, what is it?
d. Are the triangles congruent?

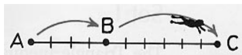


The Triangle Inequality

► Suppose a frog jumps 4 feet and then jumps 6 feet. Is it possible that it could end up 8 feet from its starting point?



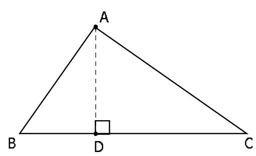
► Could the frog jump 4 feet, then jump 6 feet, and end up 12 feet from its starting point?



► **The Triangle Inequality**: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Proof of The Triangle Inequality

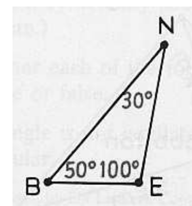
- Given: ΔABC
- Prove: BA + CA > BC
- Construct: AD ⊥ BC



- Why is BA > BD and CA > CD?
- How can you put these inequalities together using the Addition Property of Inequality?
- BD + CD equals…?
- Conclusion?

Example

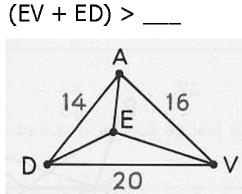
a. Can you conclude that BN < BE + EN?
b. Is it true that the length of any side of a triangle is less than the sum of the lengths of the other two sides?
c. Is it true that m∠B < m∠E + m∠N?
d. Is it true that m∠B + m∠N > m∠E?
e. Is it true that the sum of the measures of any two angles of a triangle is greater than the measure of the third angle?



Example

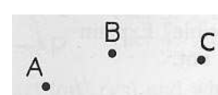
- Complete each of the inequalities:

a. ED + EA > ___
b. EA + EV > ___
c. EV + ED > ___
d. (ED + EA) + (EA + EV) + (EV + ED) > ___
e. 2ED + 2EA + 2EV > ___
f. ED + EA + EV > ___



Example – An Indirect Proof

- If AB + BC = AC, then A, B, and C are collinear.
  a. With what assumption does the proof begin?
  b. If this assumption is true, what figure do the segments $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ form?
  c. If this figure is formed, it follows that AB + BC > AC. Why?
  d. What does this conclusion contradict?
  e. What does this contradiction tell us about the assumption with which we started the proof?
  f. What conclusion follows?



Homework

- Due Tuesday 6/29
  - Read Sections 3.4 and 3.5
  - 3.3: #23-26, 34
  - 3.4: #1-32
  - 3.5: #1-18, 21-33