

Math 119 – Plane Geometry

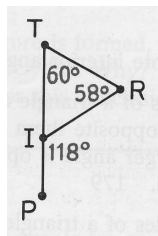
Sections 4.1 and 4.2
Quadrilaterals I
6/29/2004

Warm Up Example: True or False

- An exterior angle of a triangle is greater than any interior angle of the triangle.
- The length of each side of a triangle is less than the sum of the lengths of the other two sides.
- If two sides of a triangle are unequal, the angles opposite them are unequal.
- The smallest angle of a triangle must be opposite its shortest side.

Warm Up Example

- $\angle RIP$ is an exterior angle of $\triangle TRI$
- Find $m\angle TIR$.
 - What kind of triangle is $\triangle TRI$?
 - Which side of $\triangle TRI$ is longest?
 - Which side of $\triangle TRI$ is shortest?

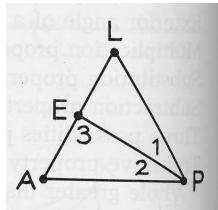


Warm Up Example: Must be True/May be True/False

► Given:

- L, E, A are collinear
- $LA < LP$
- $m\angle 1 > m\angle 2$

- a. $LE > EA$
- b. $LP = AP$
- c. $m\angle LPA < m\angle A$
- d. $\triangle LAP$ is equilateral
- e. $LE + LP > EP$
- f. LA is perpendicular to EP
- g. Angles 1 and 2 are adjacent
- h. $m\angle 3 > m\angle 1$



Warm Up Example: True/False

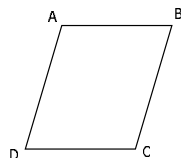
- a. If $a \neq b$, then $a > b$.
- b. If $a > c$, then $a \cdot b > c \cdot b$.
- c. The greatest angle of an isosceles triangle must be the vertex angle.
- d. If a base angle of an isosceles triangle is less than 60° , the base is the longest side.
- e. In a right triangle, the hypotenuse is the longest leg.

Quadrilaterals

► **Def:** A **quadrilateral** is a polygon that has four sides.

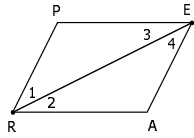
► **Def:** A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.

► Notation: $\square ABCD$



Thm 4.1.1: A diagonal of a parallelogram separates it into two congruent triangles.

- ▶ Given: \square PEAR with diagonal \overline{RE}
- ▶ Prove: $\triangle REP \cong \triangle ERA$

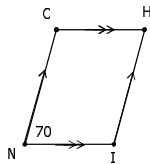


Corollaries

- ▶ **Cor 4.1.2:** Opposite angles of a parallelogram are congruent.
- ▶ **Cor 4.1.3:** Opposite sides of a parallelogram are congruent.
- ▶ **Cor 4.1.4:** Diagonals of a parallelogram bisect each other.
- ▶ **Cor 4.1.5:** Consecutive angles of a parallelogram are supplementary.

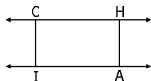
Example

- ▶ Find the measures of the missing angles



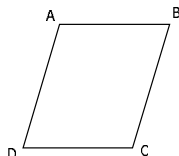
Example: Distance Between 2 Parallel Lines is Constant

- ▶ Given: $\overline{CH} \parallel \overline{IA}$
 $\overline{CI} \perp \overline{IA}$ and $\overline{HA} \perp \overline{IA}$
- ▶ Prove: $\overline{CI} \cong \overline{HA}$
- ▶ Plan: Show CHIA is a parallelogram



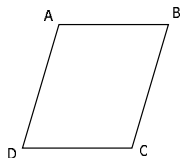
Example

- ▶ In parallelogram ABCD, $AB = 2(x + 5)$,
 $BC = x^2 + 6$, and $CD = 3(10 - x)$
- 1. Find x .
- 2. Find AD.



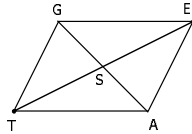
Example

- ▶ In parallelogram ABCD, $m\angle C = 3x$ and
 $m\angle D = x + 40$.
- 1. Find x .
- 2. Find $m\angle C$.
- 3. Find $m\angle D$.
- 4. Find $m\angle A$.
- 5. Find $m\angle B$.



Example

- ▶ \overline{GA} and \overline{TE} are diagonals of parallelogram GETA
- ▶ $GS = x + 20$
- ▶ $TE = 2(6 - x)$
- ▶ $SA = 2 - x$
 1. Find x .
 2. Find TS .

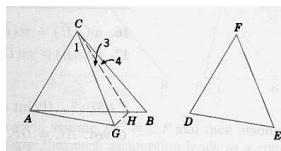


Altitudes of Parallelograms

- ▶ **Def:** An **altitude** of a parallelogram is a line segment from one vertex that is perpendicular to the opposite side (or to an extension of that side).
 - Altitude measures **distance** between 2 sides of a parallelogram
 - Measures **height** of a parallelogram
- ▶ **Ex:** Find 4 altitudes of a parallelogram.

The Hinge Theorem

- ▶ **Lemma 4.1.6:** If two sides of one triangle are congruent to two sides of a second triangle, and the included angle of the first is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.
- ▶ In terms of the picture:
 - **Given:** $AC = DF$, $BC = EF$, $m\angle C > m\angle F$
 - **Then:** $AB > DE$
- ▶ The proof of this is a result of the Triangle Inequality.
- ▶ Why do you think it is called "The Hinge Theorem"?
- ▶ Use to compare lengths of diagonals of parallelograms



Thm 4.1.7: In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.

► Use The Hinge Theorem to reason why true

► **Ex:** In parallelogram RSTV (not shown), $m\angle R = 67^\circ$.

- Find $m\angle S$.
- Which diagonal (\overline{RT} or \overline{SV}) has the greater length?

► **Ex:** In parallelogram ABCD (not shown), \overline{AC} and \overline{BD} are diagonals and $AC > BD$. Which angles are obtuse/acute?

Example

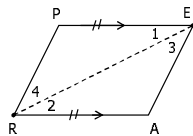
► In parallelogram MNPQ (not pictured), $m\angle M = 2(x + 10)$ and $m\angle Q = 3x - 10$. Determine which diagonal would be longer, QN or MP.

Parallelograms from Quadrilaterals

► **Thm 4.2.1:** If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

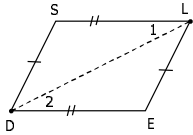
► Given: $\overline{PE} \parallel \overline{RA}$
 $\overline{PE} \cong \overline{RA}$

► Prove: PEAR is a \square

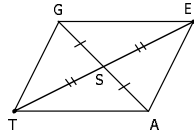


Parallelograms from Quadrilaterals

► **Thm 4.2.2:** If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram. (HW)



► **Thm 4.2.3:** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (HW)

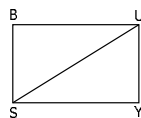


Summary: Ways to Prove a Quadrilateral is a Parallelogram

- Info on One Pair of Opposite Sides
 - Both congruent and parallel -> parallelogram
- Info on Both Pairs of Opposite Sides
 - Each pair congruent -> parallelogram
 - Both pairs of opposite sides are parallel (definition of parallelogram) -> parallelogram
- Info on Diagonals
 - Diagonals bisect each other -> parallelogram

Example: Proof

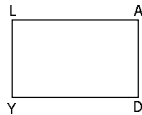
- Given: $\triangle SBU \cong \triangle UYS$
- Prove: BUSY is a parallelogram



Example: Proof

► Given: $\overline{LA} \perp \overline{AD}$ and $\overline{YD} \perp \overline{AD}$
 $\overline{LA} \cong \overline{YD}$

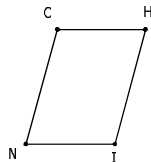
► Prove: $LADY$ is a parallelogram



Example: Proof

► Given: $CH \cong NI$ and $CN \cong HI$

► Prove: $\angle C$ and $\angle H$ are supplementary

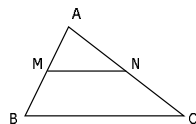


The Midsegment Theorem

► **Def:** A **midsegment** of a triangle is a line segment that joins the midpoints of two of its sides.

► **Thm 4.2.5 (The Midsegment Theorem):** A midsegment of a triangle is parallel to the third side and has a length equal to one-half the length of the third side.

- Given: ?
- Prove: ?



Proof of Midsegment Theorem

► Construct $\overline{EC} \parallel \overline{AB}$ through C

► Prove: $MN \parallel BC$

▪ Show $\triangle ANM \cong \triangle CND$

► $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

► $AN \cong CN$

▪ Show $\overline{BM} \cong \overline{CD}$

► $\overline{MB} \cong \overline{AM}$

► $\overline{AM} \cong \overline{CD}$

▪ Why is $BMDC$ a parallelogram?

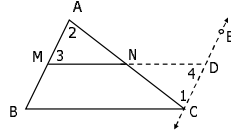
▪ Why does this prove $MN \parallel BC$?

► Prove: $MN = \frac{1}{2} BC$

▪ $MN + ND = ?$

▪ $MD = ?$

▪ How do MN and ND relate?

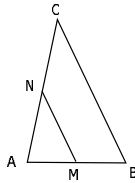


Example

► M and N are the midpoints of \overline{AB} and \overline{AC} , respectively.

a) If $BC = 12$, find MN .

b) If $MN = 15$, find BC .



Example

► N is the midpoint of \overline{AC}

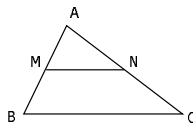
► M is the midpoint of \overline{AB}

► $MN = 2x + 1$

► $BC = 5x - 1$

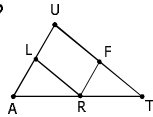
► Find x , MN and BC .

► Can you use this information to find AB ?



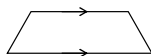
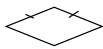
Example

- ▶ LR and FR are midsegments of $\triangle ATU$
- ▶ $m\angle A = 40$, $m\angle U = 70$, $AL = 25$, $LR = 17$
 1. What kind of quadrilateral is ULRF?
 2. Find $m\angle LRF$.
 3. Find $m\angle T$.
 4. What kind of triangle is $\triangle TUA$?
 5. Find UF .
 6. Find AU .
 7. Find AT .



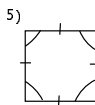
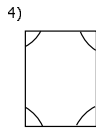
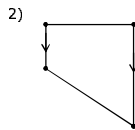
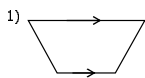
Types of Parallelograms

- ▶ A **rectangle** is a parallelogram that has a right angle.
- ▶ A **square** is a rectangle that has two congruent adjacent sides.
- ▶ A **rhombus** is a parallelogram with two congruent adjacent sides.
- ▶ A **trapezoid** is a quadrilateral with exactly two parallel sides.



Example

- ▶ Identify each of the following using the marked parts:

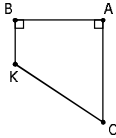


In #4, What can you conclude about the angles?

Example

- ▶ In quadrilateral $BACK$, $BK \perp BA$ and $CA \perp BA$.

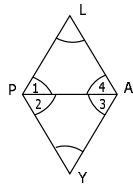
- 1) Why is $BK \parallel AC$?
- 2) What kind of quadrilateral is $BACK$?
- 3) What are $\angle C$ and $\angle K$ called with respect to transversal CK ?
- 4) Why are $\angle C$ and $\angle K$ supplementary?



Example

- ▶ In quadrilateral $PLAY$,
 $\angle L \cong \angle 1 \cong \angle 4 \cong \angle 2 \cong \angle 3 \cong \angle Y$

- ▶ What kind of triangles are $\triangle PLA$ and $\triangle PAY$?
- ▶ What kind of quadrilateral is $PLAY$?



Homework

- ▶ Due Wednesday 6/30
 - Read Sections 4.1 and 4.2
 - 4.1: #1-28
 - 4.2: #1-4, 7, 9-16, 18-20, 22-24, 27-29
