Math 119 – Plane Geometry

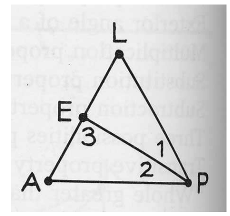
Sections 4.1 and 4.2

Quadrilaterals I

6/29/2004

Warm Up Example: Must be True/May be True/False

- Given:
  - L, E, A are collinear
  - LA < LP
  - m∠1 > m∠2



a. LE > EA
b. LP = AP
c. m∠LPA < m∠A
d. ΔLAP is equilateral
e. LE + LP > EP
f. LA is perpendicular to EP
g. Angles 1 and 2 are adjacent
h. m∠3 > m∠1

Warm Up Example: True or False

a. An exterior angle of a triangle is greater than any interior angle of the triangle.
b. The length of each side of a triangle is less than the sum of the lengths of the other two sides.
c. If two sides of a triangle are unequal, the angles opposite them are unequal.
d. The smallest angle of a triangle must be opposite its shortest side.

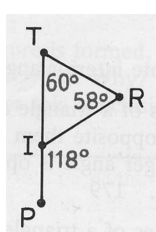
Warm Up Example: True/False

a. If $a \neq b$, then $a > b$.
b. If $a > c$, then $a \cdot b > c \cdot b$.
c. The greatest angle of an isosceles triangle must be the vertex angle.
d. If a base angle of an isosceles triangle is less than 60, the base is the longest side.
e. In a right triangle, the hypotenuse is the longest leg.

Warm Up Example

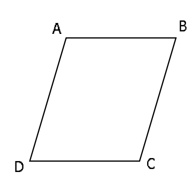
- ∠RIP is an exterior angle of ΔTRI

a. Find m∠TIR.
b. What kind of triangle is ΔTRI?
c. Which side of ΔTRI is longest?
d. Which side of ΔTRI is shortest?



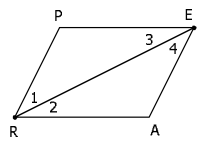
Quadrilaterals

- **Def**: A **quadrilateral** is a polygon that has four sides.
- **Def**: A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.
- Notation: ▱ ABCD



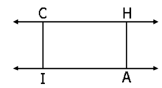
Thm 4.1.1: A diagonal of a parallelogram separates it into two congruent triangles.

- Given: ▱PEAR with diagonal $\overline{RE}$
- Prove: $\triangle REP \cong \triangle ERA$



Example: Distance Between 2 Parallel Lines is Constant

- Given: $\overleftrightarrow{CH} \parallel \overleftrightarrow{IA}$
  $\overline{CI} \perp \overleftrightarrow{IA}$ and $\overline{HA} \perp \overleftrightarrow{IA}$
- Prove: $\overline{CI} \cong \overline{HA}$
- Plan: Show CHIA is a parallelogram

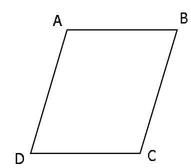


Corollaries

- **Cor 4.1.2**: Opposite angles of a parallelogram are congruent.
- **Cor 4.1.3**: Opposite sides of a parallelogram are congruent.
- **Cor 4.1.4**: Diagonals of a parallelogram bisect each other.
- **Cor 4.1.5**: Consecutive angles of a parallelogram are supplementary.

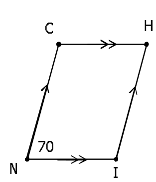
Example

- In parallelogram ABCD, AB = $2(x + 5)$, BC = $x^2 + 6$, and CD = $3(10 - x)$
  1. Find x.
  2. Find AD.



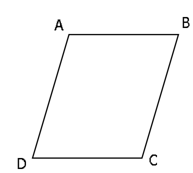
Example

- Find the measures of the missing angles



Example

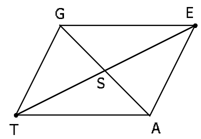
- In parallelogram ABCD, $m\angle C = 3x$ and $m\angle D = x + 40$.
  1. Find x.
  2. Find $m\angle C$.
  3. Find $m\angle D$.
  4. Find $m\angle A$.
  5. Find $m\angle B$.



Example

- $\overline{GA}$ and $\overline{TE}$ are diagonals of parallelogram GETA
- GS = x + 20
- TE = 2(6 – x)
- SA = 2 – x

1. Find x.
2. Find TS.



Thm 4.1.7: In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.

- Use The Hinge Theorem to reason why true
- **Ex**: In parallelogram RSTV (not shown), m∠R = 67°.
  - Find m∠S.
  - Which diagonal ($\overline{RT}$ or $\overline{SV}$) has the greater length?
- **Ex**: In parallelogram ABCD (not shown), $\overline{AC}$ and $\overline{BD}$ are diagonals and AC > BD. Which angles are obtuse/acute?

Altitudes of Parallelograms

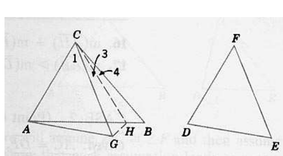
- **Def**: An **altitude** of a parallelogram is a line segment from one vertex that is perpendicular to the opposite side (or to an extension of that side).
  - Altitude measures **distance** between 2 sides of a parallelogram
  - Measures **height** of a parallelogram
- **Ex**: Find 4 altitudes of a parallelogram.

Example

- In parallelogram MNPQ (not pictured), m∠M = 2(x + 10) and m∠Q = 3x – 10. Determine which diagonal would be longer, $\overline{QN}$ or $\overline{MP}$.

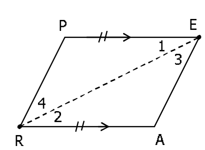
The Hinge Theorem

- **Lemma 4.1.6**: If two sides of one triangle are congruent to two sides of a second triangle, and the included angle of the first is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.
- In terms of the picture:
  - **Given**: AC = DF, BC = EF, m∠C > m∠F
  - **Then**: AB > DE
- The proof of this is a result of the Triangle Inequality.
- Why do you think it is called "The Hinge Theorem"?
- Use to compare lengths of diagonals of parallelograms



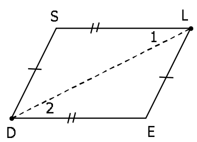
Parallelograms from Quadrilaterals

- **Thm 4.2.1**: If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.
- Given: $\overline{PE} \parallel \overline{RA}$, $\overline{PE} \cong \overline{RA}$
- Prove: PEAR is a ▱

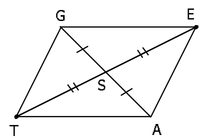


Parallelograms from Quadrilaterals

- **Thm 4.2.2**: If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram. (HW)

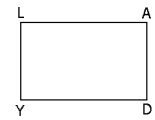


- **Thm 4.2.3**: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (HW)



Example: Proof

- Given: $\overline{LA} \perp \overline{AD}$ and $\overline{YD} \perp \overline{AD}$
  $\overline{LA} \cong \overline{YD}$
- Prove: LADY is a parallelogram

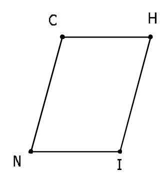


Summary: Ways to Prove a Quadrilateral is a Parallelogram

- Info on One Pair of Opposite Sides
  - Both congruent and parallel -> parallelogram
- Info on Both Pairs of Opposite Sides
  - Each pair congruent -> parallelogram
  - Both pairs of opposite sides are parallel (definition of parallelogram) -> parallelogram
- Info on Diagonals
  - Diagonals bisect each other -> parallelogram

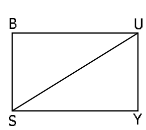
Example: Proof

- Given: CH ≅ NI and CN ≅ HI
- Prove: ∠C and ∠H are supplementary



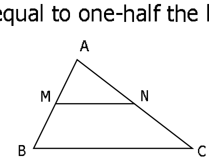
Example: Proof

- Given: ΔSBU ≅ ΔUYS
- Prove: BUSY is a parallelogram



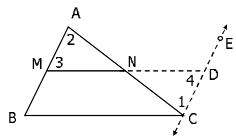
The Midsegment Theorem

- **Def**: A **midsegment** of a triangle is a line segment that joins the midpoints of two of its sides.
- **Thm 4.2.5 (The Midsegment Theorem)**: A midsegment of a triangle is parallel to the third side and has a length equal to one-half the length of the third side.
  - Given: ?
  - Prove: ?



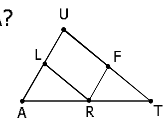
Proof of Midsegment Theorem

- Construct $\overleftrightarrow{EC} \parallel \overline{AB}$ through C
- Prove: $\overline{MN} \parallel \overline{BC}$
  - Show $\Delta ANM \cong \Delta CND$
    - $\angle 1 \cong 2$, $\angle 3 \cong \angle 4$
    - $\overline{AN} \cong \overline{NC}$
  - Show $\overline{BM} \cong \overline{CD}$
    - $\overline{MB} \cong \overline{AM}$
    - $\overline{AM} \cong \overline{CD}$
  - Why is BMDC a parallelogram?
  - Why does this prove $\overline{MN} \parallel \overline{BC}$?
- Prove: MN = ½ BC
  - MN + ND = ?
  - MD = ?
  - How do MN and ND relate?



Example

- LR and FR are midsegments of ΔATU
- $m\angle A = 40$, $m\angle U = 70$, AL = 25, LR = 17
  1. What kind of quadrilateral is ULRF?
  2. Find $m\angle LRF$.
  3. Find $m\angle T$.
  4. What kind of triangle is ΔTUA?
  5. Find UF.
  6. Find AU.
  7. Find AT.

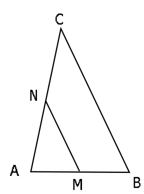


Example

- M and N are the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively.

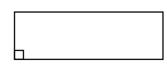
a) If BC = 12, find MN.

b) If MN = 15, find BC.

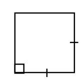


Types of Parallelograms

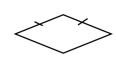
- A **rectangle** is a parallelogram that has a right angle.



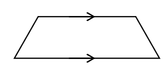
- A **square** is a rectangle that has two congruent adjacent sides.



- A **rhombus** is a parallelogram with two congruent adjacent sides.

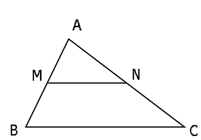


- A **trapezoid** is a quadrilateral with exactly two parallel sides.



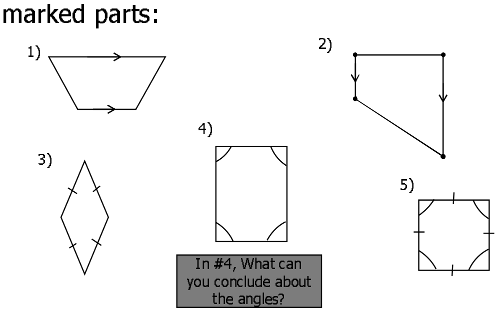
Example

- N is the midpoint of $\overline{AC}$
- M is the midpoint of $\overline{AB}$
- MN = 2x + 1
- BC = 5x – 1
- Find x, MN and BC.
- Can you use this information to find AB?



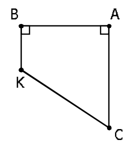
Example

- Identify each of the following using the marked parts:



Example

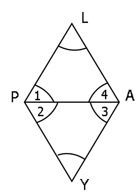
- In quadrilateral BACK, BK ⊥ BA and CA ⊥ BA.



1) Why is BK ∥ AC?
2) What kind of quadrilateral is BACK?
3) What are ∠C and ∠K called with respect to transversal CK?
4) Why are ∠C and ∠K supplementary?

Example

- In quadrilateral PLAY, ∠L ≅ ∠1 ≅ ∠4 ≅ ∠2 ≅ ∠3 ≅ ∠Y



- What kind of triangles are ΔPLA and ΔPAY?
- What kind of quadrilateral is PLAY?

Homework

- Due Wednesday 6/30
  - Read Sections 4.1 and 4.2
  - 4.1: #1-28
  - 4.2: #1-4, 7, 9-16, 18-20, 22-24, 27-29