

# Math 119 – Plane Geometry

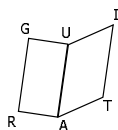
Sections 4.3 and 4.4  
 Quadrilaterals II  
 6/30/2004

## Warm Up Example

- Complete the following statements:
1. The opposite sides of a parallelogram are \_\_\_ and \_\_\_.
  2. The consecutive angles of a parallelogram are \_\_\_.
  3. The opposite angles of a parallelogram are \_\_\_.
  4. The diagonals of a parallelogram are \_\_\_.

## Warm Up Example

- Given: GUAR and UITA are parallelograms  
 ► Prove:  $\overline{GR} \cong \overline{IT}$



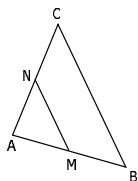
## Warm Up Example: True/False

1. If a quadrilateral is a parallelogram, then at least two opposite sides are parallel.
2. A quadrilateral is a parallelogram if at least two opposite sides are parallel.
3. If a quadrilateral is a parallelogram, then its diagonals bisect each other.
4. If a quadrilateral is not a parallelogram, then its diagonals do not bisect each other.
5. If a quadrilateral is a parallelogram, then it is equilateral.
6. A quadrilateral is a parallelogram if it is equilateral.

## Warm Up Example

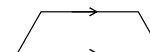
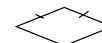
- MN is a midsegment of  $\triangle ABC$ .  
 $m\angle C = m\angle B = 60$ .  $CB = 16$ .

1. Find  $m\angle A$ .
2. What kind of triangle is  $\triangle ABC$ ?
3. What kind of triangle is  $\triangle AMN$ ?
4. What kind of quadrilateral is MNCB?
5. Find MN.
6. Find RA.



## Recall: Types of Parallelograms

- A **rectangle** is a parallelogram that has \_\_\_\_.
- A **square** is a rectangle that has \_\_\_\_.
- A **rhombus** is a parallelogram with \_\_\_\_.
- A **trapezoid** is a quadrilateral with \_\_\_\_.



## Theorems About Rectangles

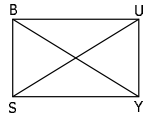
► **Cor 4.3.1:** All angles of a rectangle are right angles.

► **Thm 4.3.2:** The diagonals of a rectangle are congruent.

▪ Given: Rectangle BUSY with diagonals  $\overline{SU}$  and  $\overline{BY}$

▪ Prove:  $\overline{SU} \cong \overline{BY}$

▪ Plan: Examine  $\triangle SBU$  and  $\triangle YUB$

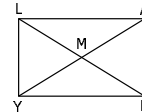


## Example

► Quadrilateral LADY is a rectangle.  $\overline{LD}$  and  $\overline{AY}$  are its diagonals.

1. What kind of triangle is  $\triangle ADY$ ? Why?

2. What kind of triangle is  $\triangle YMD$ ? Why?

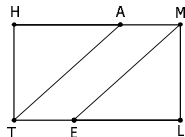


## Example

► Given: HMLT is a rectangle

$\overline{HA} \cong \overline{EL}$

► Prove:  $\overline{TA} \cong \overline{EM}$



## Theorems About Squares

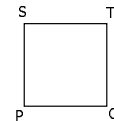
► **Cor 4.3.3:** All sides of a square are congruent.

► **Ex:** Quadrilateral STOP is a square.

1. Why is STOP a rhombus?

2. Why is STOP a parallelogram?

3. Why is STOP a rectangle?



## Theorems About Rhombuses

► **Cor 4.3.4:** All sides of a rhombus are congruent.

► **Thm 4.3.5:** The diagonals of a rhombus are perpendicular.

▪ Given: Rhombus ABCD with diagonals  $\overline{AC}$  and  $\overline{DB}$

▪ Prove:  $\overline{AC} \perp \overline{DB}$

▪ Plan: Examine  $\triangle ABE$  and  $\triangle CBE$

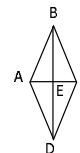


## Example

► Quadrilateral ABCD is a rhombus and  $\overline{BD}$  and  $\overline{AC}$  are its diagonals.

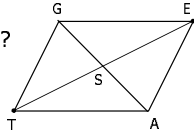
1. What kind of triangle is  $\triangle ABC$ ?

2. What kind of triangle is  $\triangle ABE$ ?



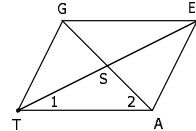
### Example

- ▶ In quadrilateral GATE,  $\overline{GA}$  and  $\overline{ET}$  bisect each other.
  1. What kind of quadrilateral is GATE? Why?
  2. Does it follow that  $\overline{GA} \perp \overline{ET}$ ?
  3. Does it follow that  $\overline{GA} \cong \overline{ET}$ ?



### Example

- ▶ Given: GATE is a rhombus with diagonals  $\overline{GA}$  and  $\overline{ET}$
- ▶ Prove:  $\angle 1$  and  $\angle 2$  are complementary

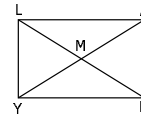


### Pythagorean Theorem Revisited

- ▶ In a right triangle with hypotenuse of length  $c$  and legs of lengths  $a$  and  $b$ ,  $c^2 = a^2 + b^2$ .
- ▶ **Ex:** What is the length of the diagonal in a rectangle whose sides measure 3 ft and 4 ft?
- ▶ **Ex:** What is the length of each side of a rhombus whose diagonals measure 10 cm and 24 cm?

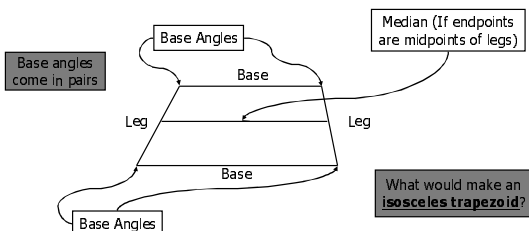
### Example

- ▶ Quadrilateral LADY is a rectangle with diagonals  $\overline{LD}$  and  $\overline{AY}$ .
- ▶  $LY = 10$  and  $YD = 24$ 
  1. Find LD.
  2. Find LM.



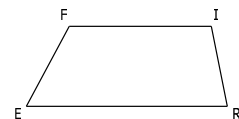
### The Trapezoid

- ▶ A **trapezoid** is a quadrilateral with exactly two parallel sides.



### Example

- ▶ Quadrilateral FIRE is a trapezoid with  $\overline{FI} \parallel \overline{ER}$ 
  1. Which sides are the bases?
  2. Which sides are the legs?
  3. Does FIRE appear to be isosceles?
- ▶ Suppose  $m\angle F = 107$  and  $m\angle R = 58$ . Find the measures of the other angles.

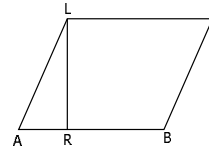


### Example: True/False

1. If a quadrilateral is a trapezoid, its base angles are equal.
2. A trapezoid can have three equal sides.
3. A trapezoid can have four equal angles.
4. A trapezoid can have three right angles.
5. If the diagonals of a quadrilateral bisect each other, it is not a trapezoid.

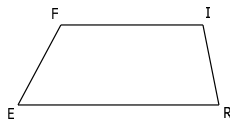
### Example

- ▶ Given: LIBR is a trapezoid with bases  $\overline{LI}$  and  $\overline{RB}$   
 $\overline{LI} \cong \overline{BA}$
- ▶ Prove: LIBA is a parallelogram



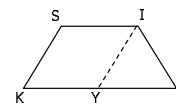
### Altitudes of Trapezoids

- ▶ An **altitude** of a trapezoid is a line segment from one vertex of one base of the trapezoid perpendicular to the opposite base (or to an extension of that base).
- ▶ **Ex:** Draw altitudes from each of the four vertices:



### Isosceles Trapezoid Theorems

- ▶ **Thm 4.4.1:** The base angles of an isosceles trapezoid are congruent.
  - Given: Trapezoid SILK  
 $SK \cong IL$  and  $\overline{SI} \parallel \overline{KL}$
  - Prove:  $\angle S \cong \angle I$  and  $\angle K \cong \angle L$
  - Plan:
    - ▶ Construct Y so that  $\overline{IY} \parallel \overline{SK}$
    - ▶ Get  $\angle K \cong \angle L$
    - ▶ Then get  $\angle S \cong \angle I$

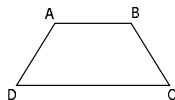


- ▶ **Cor 4.4.2:** The diagonals of an isosceles trapezoid are congruent. (SAS; cpctc)

### Example

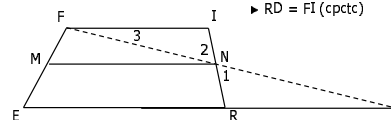
- ▶ Given isosceles trapezoid ABCD with  $\overline{AB} \parallel \overline{DC}$

  1. Find the measures of the angles of ABCD if  $m\angle A = 12x + 30$  and  $m\angle B = 10x + 46$ .
  2. Using the result from part (1), find the length of each diagonal if  $AC = 2x - 5$ .



### Theorems For Medians of Trapezoids

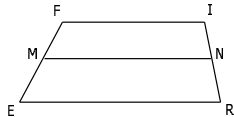
- ▶ **Thm 4.4.3:** The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.
- ▶ **Thm 4.4.4:** The median of a trapezoid is parallel to each base.
- ▶ **Given:** Trapezoid FIRE  
 median  $\overline{MN}$   
 $\overline{FI} \parallel \overline{ER}$
- ▶ **Prove:**  $\overline{MN} \parallel \overline{ER}$  and  $MN = \frac{1}{2}(FI + ER)$
- ▶ **Plan**
  - Construct line  $\overline{FN}$
  - $\triangle DNR \cong \triangle FNI$  by AAS
  - Get  $\overline{MN}$  as midsegment of  $\triangle EFD$ 
    - ▶ 4.4.3 Follows
    - ▶  $MN = \frac{1}{2}(ED) = \frac{1}{2}(ER + RD)$
    - ▶  $RD = FI$  (cpctc)



### Example

► In trapezoid FIRE,  $\overline{FI} \parallel \overline{ER}$  and M and N are midpoints of  $\overline{FE}$  and  $\overline{IR}$ , respectively.

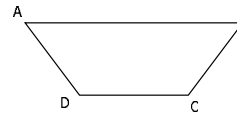
1. Find the length of median  $\overline{MN}$  if  $FI = 12$  and  $ER = 18$ .
2. Find MN, FI, and ER if  $FI = 2x$ ,  $MN = 3x - 5$ , and  $ER = 2x + 10$ .



### Proving Quadrilaterals Trapezoids

► **Thm 4.4.5:** If two of three consecutive angles of a quadrilateral are supplementary, the quadrilateral is a trapezoid.

- What kind of a figure would it be if 2 of 2 consecutive angles were supplementary?



### Proving Isosceles Trapezoids

► **Thm 4.4.6:** If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

► **Thm 4.4.7:** If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

### Transversals

► **Thm 4.4.8:** If three (or more) parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any transversal.

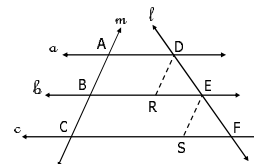
► Given: Parallel lines  $a, b, c$

$$\overline{AB} \cong \overline{BC}$$

► Prove:  $\overline{DE} \cong \overline{EF}$

► Plan:

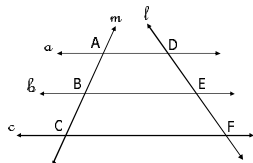
- Draw  $\overline{DR}$  and  $\overline{ES}$  parallel to  $\ell$
- Show  $\triangle DER \cong \triangle EFS$  by AAS



### Example

► Given: Parallel lines  $a, b, c$ .

► If  $AB = BC = 7.2$  and  $DE = 8.4$ , find EF.



### Homework

► Due Thursday 7/1

- Read Sections 4.3 and 4.4
- 4.3: #1-31
- 4.4: #1-18, 21-23, 25, 26

► Reminder: Exam 3 – Tuesday, July 6