Math 119 – Plane Geometry

Sections 5.1 and 5.2
Similarity I
7/1/2004

Review Examples: True/False

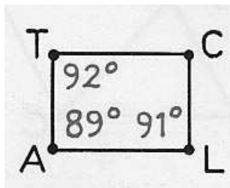
1. If both pairs of opposite angles in a quadrilateral are congruent, the quadrilateral must be a parallelogram.
2. One of the diagonals of a parallelogram divides it into two congruent triangles.
3. If one of the diagonals of a quadrilateral divides it into two congruent triangles, the quadrilateral must be a parallelogram.
4. All squares are rhombuses.

Review Examples: True/False

5. All parallelograms are rectangles.
6. If two consecutive sides of a parallelogram are congruent, the parallelogram must be a rhombus.
7. All four angles of a trapezoid can have different measures.
8. The opposite angles of an isosceles trapezoid are supplementary.
9. The diagonals of an isosceles triangle bisect each other.

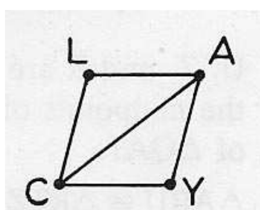
Warm Up Example

- What kind of quadrilateral is TALC? Why?



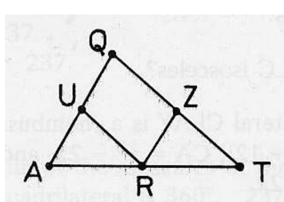
Warm Up Example

- Quadrilateral CLAY is a rhombus.
- CL = 3(x + 12)
- CA = $x^2 - 25$
- CY = 1 – 2x
- Find x.



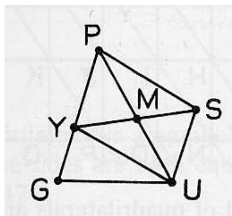
Warm Up Example

- Given:
  - U, Z, and R are the midpoints of the sides of triangle QAT
- Prove:
  - QURZ is a parallelogram



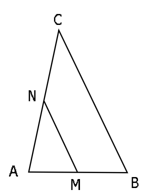
Warm Up Example

- Given:
  - In quadrilateral GPSU, $\overline{PU}$ and $\overline{YS}$ bisect each other
  - $\overline{PS} \simeq \overline{GU}$
- Prove:
  - $\overline{GU} \simeq \overline{YU}$



Warm Up Example

- Given: ΔABC with $\overline{NM} \parallel \overline{CB}$
  $\overline{CN} \cong \overline{MB}$
- Prove: ΔABC is isosceles



Ratio

- A **ratio** is the comparison of two numbers by their indicated quotient.
- Express as fractions
  - The **ratio** of *a* to *b* is the number a/b
- **Ex:** Express each ratio in lowest terms:
  - 15 to 12
  - 3 in. to 7 in.
  - 2 ft to 1 yd.

Warning! Watch out for units…

Proportion

- A **proportion** is an equality of two ratios.
- Written: *a/b = c/d*
- Said: "a is to b as c is to d"
- *a* is the **first term**; *b* is the **second term**; *c* is the **third term**; *d* is the **fourth term**
- *d* is also called the **fourth proportional**
- *a* and *d* are also called the **extremes**
- *c* and *b* are also called the **means**
- **Ex:** Name the parts of 2/6 = 3/9

Means-Extremes Property

- In any proportion, the product of the extremes equals the product of the means.
  - Rewrite this statement in symbols
- AKA: Cross Multiplication
- **Ex:** Use this property to determine if the two fractions 7/12 and 21/34 are equal.

Solving Proportions

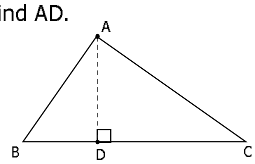
1. **Ex**: Solve x/5 = 24/20
2. **Ex**: Solve (n + 3)/n = 4/3
3. **Ex**: Solve (x + 3)/3 = 9/(x – 3)
4. **Ex**: Solve 3/x = x/2
5. **Ex**: Solve (x + 2)/5 = 4/(x – 1)

Example

- Two complementary angles are in the ratio 2 to 3. Find the measure of each angle.
  - Method 1: Set up proportion
  - Method 2: 2 to 3 means 2x, 3x

Geometric Mean

- If a/b = b/c, then b is the **geometric mean** of a and c.
  - **Ex**: 6 and -6 are the geometric means of 4 and 9
  - **Ex**: AD is the geometric mean of BD and DC. If BC = 10 and BD = 4, find AD.



Property 2: In a proportion, the means or the extremes or both may be interchanged.

- **Ex**: Given the proportion 3/7 = 18/42, write 3 new proportions that follow.
- In any of the equivalent forms, what equation does the Means-Extremes Property produce?
  - How do these equations relate to each other?

Property 3: If a/b = c/d, then (a + b)/b = (c + d)/d and (a – b)/b = (c – d)/d.

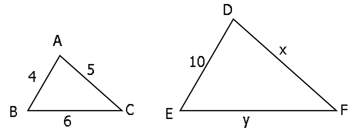
- Prove by considering a/b = c/d and add (or subtract) 1 to both sides
- **Ex**: Given the proportion 3/7 = 18/42, write (and verify) 2 new proportions that follow.

Extended Ratios and Proportions

- An **extended ratio** compares more than two quantities
  - a:b:c: …
  - Variable expressions: ax, bx, cx, …
  - **Ex**: Suppose the perimeter of a quadrilateral is 70 and the lengths of the sides are in the ratio 2:3:4:5. Find the measure of each side.
- **Extended proportions** are in the form a/b = c/d = e/f = …

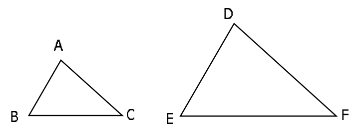
Example

- AB/DE = AC/DF = BC/EF
- Find DF and EF



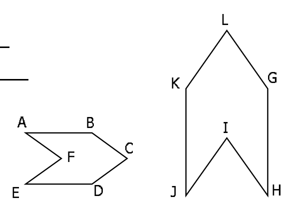
Similar Polygons

- Two polygons are **similar** if, and only if,
  - All pairs of corresponding angles are congruent
  - All pairs of corresponding sides are proportional
- Denoted: ~
- **Similar** means "has the same shape"
- **Ex:** Indicate criteria for ΔABC ~ ΔDEF



Example

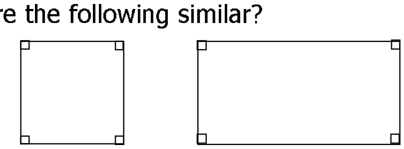
- Complete the following correspondence:
  - ABCDE ↔ ____
- Complete the following equations:
  - m∠A = m∠____
  - BC/KL = DE/____
  - EF/____ = AF/____



Example

- Are the following similar?

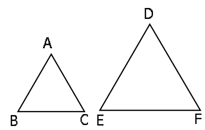
1.



2. Any two isosceles triangles?
3. Any two rectangles?
4. Any two regular pentagons?
5. Any two squares?

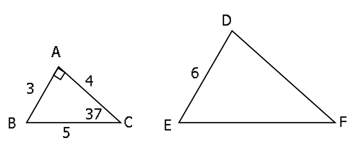
Example

- The two triangles are equilateral
  - Why is AB = BC = CA and DE = EF = FD?
  - Why is AB/DE = BC/EF = CA/FD?
  - Why are the corresponding angles congruent?
  - Why are the triangles similar?



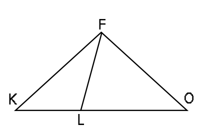
Example

- If ΔABC ~ ΔDEF, find the remaining parts of the triangles.



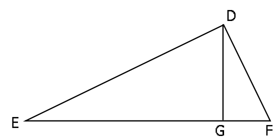
Example

- Given: ΔFLK ~ ΔKFO
- Prove: ΔFLK is isosceles



Example

- ► Given: ΔEGD ~ ΔDGF
- ► Prove: DG is the geometric mean between EG and GF

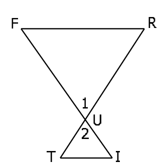


A.A. Similarity Theorem

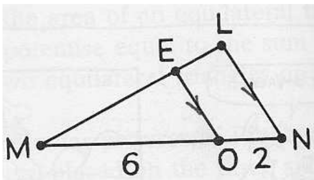
- ► **Post 15**: If the three angles of one triangle are congruent to the three angles of a second triangle, then the triangles are similar (AAA).
- ► **Cor 5.2.1**: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar (AA).

Example

- ► Given: $\overline{FR} \parallel \overline{TI}$
- ► Prove: ΔFRU ~ ΔITU

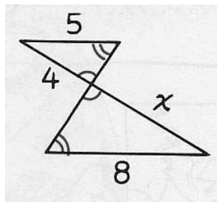


Example



▶ $\overline{EO}$ || $\overline{LN}$. Find EO/LN.

▶ Find x.



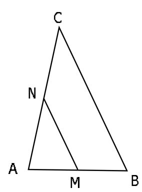
CSSTP: Corresponding sides of similar triangles are proportional.

▶ Example:

▶ Given: ∠AMN ≅ ∠ABC

▶ Prove: MN/BC = AN/AC

Note: Corresponding sides belong in the same ratio



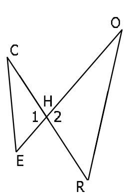
Example

▶ Given: ∠C ≅ ∠O

▶ Prove: EH · OR = RH · CE

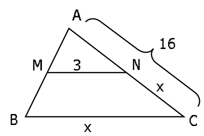
▶ Plan:

1. Show ΔCHE ~ ΔOHR by AA
2. Use CSSTP
3. Use Means-Extremes Property



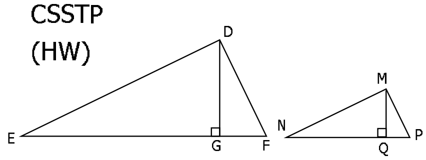
Example

- Suppose ∠AMN ≅ ∠B. If MN = 3, AC = 16, and NC = BC, find BC.



Thm 5.2.2 The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.

- Given: $\Delta DEF \sim \Delta MNP$; $\overline{DG}$ and $\overline{MQ}$ are altitudes
- Prove: DG/MQ = DE/MN
- Plan: Show $\Delta DEG \sim \Delta MNQ$ using AA; CSSTP (HW)



Homework

- Due Tuesday 7/6
  - Read Sections 5.1 and 5.2
  - 5.1: #1-14, 17-22, 24-30
  - 5.2: #1-4, 13-19, 21-35, 40, 42
- Exam 3 – Tuesday, July 6
  - Covering Chapters 3 and 4; Sections 5.1, 5.2
- Suggested Preparation:
  - Do HW above
  - Chapter 3 Review: #1-29
  - Chapter 4 Review: #1-9, 11, 13-33