

Math 119 – Plane Geometry

Sections 5.1 and 5.2
Similarity I
7/1/2004

Review Examples: True/False

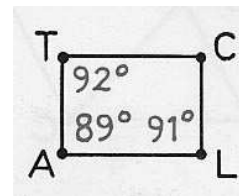
1. If both pairs of opposite angles in a quadrilateral are congruent, the quadrilateral must be a parallelogram.
2. One of the diagonals of a parallelogram divides it into two congruent triangles.
3. If one of the diagonals of a quadrilateral divides it into two congruent triangles, the quadrilateral must be a parallelogram.
4. All squares are rhombuses.

Review Examples: True/False

5. All parallelograms are rectangles.
6. If two consecutive sides of a parallelogram are congruent, the parallelogram must be a rhombus.
7. All four angles of a trapezoid can have different measures.
8. The opposite angles of an isosceles trapezoid are supplementary.
9. The diagonals of an isosceles triangle bisect each other.

Warm Up Example

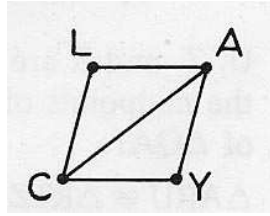
- What kind of quadrilateral is TALC? Why?



Warm Up Example

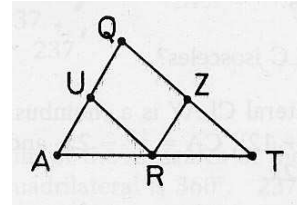
- ▶ Quadrilateral CLAY is a rhombus.
- ▶ $CL = 3(x + 12)$
- ▶ $CA = x^2 - 25$
- ▶ $CY = 1 - 2x$

- ▶ Find x .



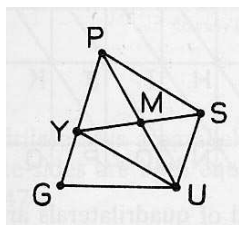
Warm Up Example

- ▶ Given:
 - $U, Z,$ and R are the midpoints of the sides of triangle QAT
- ▶ Prove:
 - QRZ is a parallelogram



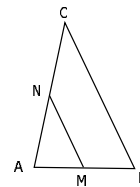
Warm Up Example

- ▶ Given:
 - In quadrilateral $GPSU$, \overline{PU} and \overline{YS} bisect each other
 - $\overline{PS} \cong \overline{GU}$
- ▶ Prove:
 - $\overline{GU} \cong \overline{YU}$



Warm Up Example

- ▶ Given: $\triangle ABC$ with $\overline{NM} \parallel \overline{CB}$
 $\overline{CN} \cong \overline{MB}$
- ▶ Prove: $\triangle ABC$ is isosceles



Ratio

- ▶ A **ratio** is the comparison of two numbers by their indicated quotient.
- ▶ Express as fractions
 - The **ratio** of a to b is the number a/b
- ▶ **Ex:** Express each ratio in lowest terms:
 - 15 to 12
 - 3 in. to 7 in.
 - 2 ft to 1 yd.

Warning!
Watch out for
units...

Proportion

- ▶ A **proportion** is an equality of two ratios.
- ▶ Written: $a/b = c/d$
- ▶ Said: "a is to b as c is to d"

- ▶ a is the **first term**; b is the **second term**; c is the **third term**; d is the **fourth term**
- ▶ d is also called the **fourth proportional**

- ▶ a and d are also called the **extremes**
- ▶ c and b are also called the **means**

- ▶ **Ex:** Name the parts of $2/6 = 3/9$

Means-Extremes Property

- ▶ In any proportion, the product of the extremes equals the product of the means.
 - Rewrite this statement in symbols
- ▶ AKA: Cross Multiplication

- ▶ **Ex:** Use this property to determine if the two fractions $7/12$ and $21/34$ are equal.

Solving Proportions

1. **Ex:** Solve $x/5 = 24/20$
2. **Ex:** Solve $(n + 3)/n = 4/3$
3. **Ex:** Solve $(x + 3)/3 = 9/(x - 3)$
4. **Ex:** Solve $3/x = x/2$
5. **Ex:** Solve $(x + 2)/5 = 4/(x - 1)$

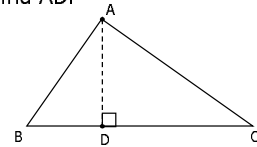
Example

- ▶ Two complementary angles are in the ratio 2 to 3. Find the measure of each angle.
 - Method 1: Set up proportion

 - Method 2: 2 to 3 means $2x$, $3x$

Geometric Mean

- ▶ If $a/b = b/c$, then b is the **geometric mean** of a and c .
 - **Ex:** 6 and -6 are the geometric means of 4 and 9
 - **Ex:** AD is the geometric mean of BD and DC. If $BC = 10$ and $BD = 4$, find AD.



Property 2: In a proportion, the means or the extremes or both may be interchanged.

- ▶ **Ex:** Given the proportion $3/7 = 18/42$, write 3 new proportions that follow.

- ▶ In any of the equivalent forms, what equation does the Means-Extremes Property produce?
 - How do these equations relate to each other?

Property 3: If $a/b = c/d$, then $(a + b)/b = (c + d)/d$ and $(a - b)/b = (c - d)/d$.

- ▶ Prove by considering $a/b = c/d$ and add (or subtract) 1 to both sides

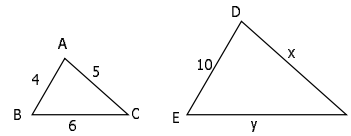
- ▶ **Ex:** Given the proportion $3/7 = 18/42$, write (and verify) 2 new proportions that follow.

Extended Ratios and Proportions

- ▶ An **extended ratio** compares more than two quantities
 - $a:b:c$: ...
 - Variable expressions: ax, bx, cx, \dots
 - **Ex:** Suppose the perimeter of a quadrilateral is 70 and the lengths of the sides are in the ratio 2:3:4:5. Find the measure of each side.
- ▶ **Extended proportions** are in the form $a/b = c/d = e/f = \dots$

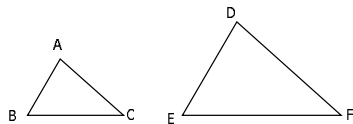
Example

- ▶ $AB/DE = AC/DF = BC/EF$
- ▶ Find DF and EF



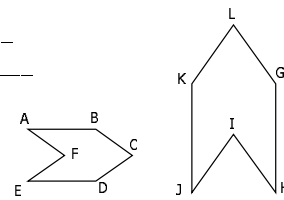
Similar Polygons

- ▶ Two polygons are **similar** if, and only if,
 - All pairs of corresponding angles are congruent
 - All pairs of corresponding sides are proportional
- ▶ Denoted: \sim
- ▶ **Similar** means "has the same shape"
- ▶ **Ex:** Indicate criteria for $\triangle ABC \sim \triangle DEF$



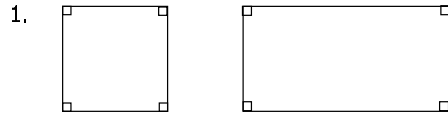
Example

- ▶ Complete the following correspondence:
 - $ABCDE \leftrightarrow \underline{\hspace{2cm}}$
- ▶ Complete the following equations:
 - $m\angle A = m\angle \underline{\hspace{2cm}}$
 - $BC/KL = DE/\underline{\hspace{2cm}}$
 - $EF/\underline{\hspace{2cm}} = AF/\underline{\hspace{2cm}}$



Example

► Are the following similar?

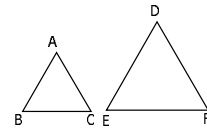


2. Any two isosceles triangles?
3. Any two rectangles?
4. Any two regular pentagons?
5. Any two squares?

Example

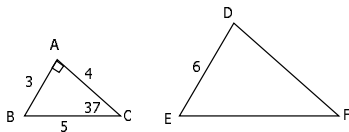
► The two triangles are equilateral

- Why is $AB = BC = CA$ and $DE = EF = FD$?
- Why is $AB/DE = BC/EF = CA/FD$?
- Why are the corresponding angles congruent?
- Why are the triangles similar?



Example

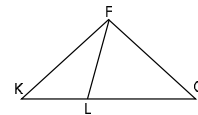
► If $\triangle ABC \sim \triangle DEF$, find the remaining parts of the triangles.



Example

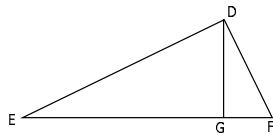
► Given: $\triangle FLK \sim \triangle KFO$

► Prove: $\triangle FLK$ is isosceles



Example

- ▶ Given: $\triangle EGD \sim \triangle DGF$
- ▶ Prove: DG is the geometric mean between EG and GF

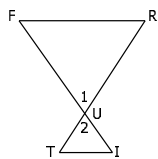


A.A. Similarity Theorem

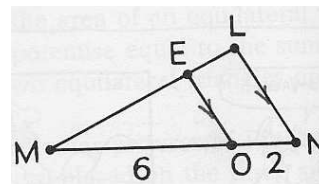
- ▶ **Post 15:** If the three angles of one triangle are congruent to the three angles of a second triangle, then the triangles are similar (AAA).
- ▶ **Cor 5.2.1:** If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar (AA).

Example

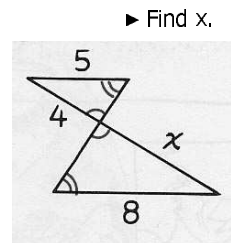
- ▶ Given: $\overline{FR} \parallel \overline{TI}$
- ▶ Prove: $\triangle FRU \sim \triangle ITU$



Example



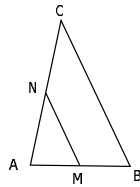
- ▶ $\overline{EO} \parallel \overline{LN}$. Find EO/LN.



CSSTP: Corresponding sides of similar triangles are proportional.

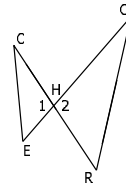
- ▶ Example:
- ▶ Given: $\angle AMN \cong \angle ABC$
- ▶ Prove: $MN/BC = AN/AC$

Note: Corresponding sides belong in the same ratio



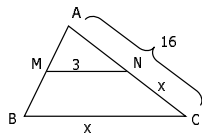
Example

- ▶ Given: $\angle C \cong \angle O$
- ▶ Prove: $EH \cdot OR = RH \cdot CE$
- ▶ Plan:
 1. Show $\triangle CHE \sim \triangle OHR$ by AA
 2. Use CSSTP
 3. Use Means-Extremes Property



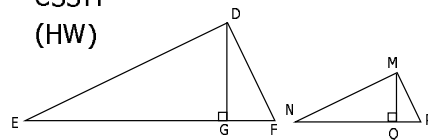
Example

- ▶ Suppose $\angle AMN \cong \angle B$. If $MN = 3$, $AC = 16$, and $NC = BC$, find BC .



Thm 5.2.2 The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.

- ▶ Given: $\triangle DEF \sim \triangle MNP$;
DG and MQ are altitudes
- ▶ Prove: $DG/MQ = DE/MN$
- ▶ Plan: Show $\triangle DEG \sim \triangle MNQ$ using AA;
CSSTP
(HW)



Homework

- ▶ Due Tuesday 7/6
 - Read Sections 5.1 and 5.2
 - 5.1: #1-14, 17-22, 24-30
 - 5.2: #1-4, 13-19, 21-35, 40, 42

- ▶ Exam 3 – Tuesday, July 6
 - Covering Chapters 3 and 4; Sections 5.1, 5.2
- ▶ Suggested Preparation:
 - Do HW above
 - Chapter 3 Review: #1-29
 - Chapter 4 Review: #1-9, 11, 13-33