Math 119 – Plane Geometry

Sections 6.1 and 6.2

Circles I

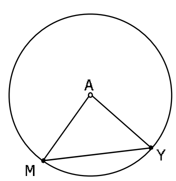
7/8/2004

Circle Definitions

- A **circle** is the set of all points *in a plane* that are at a fixed distance from a given point (its **center**).
  - Denoted: ⊙P, where P is the center
- A **radius** is a segment that joins the center to a point on the circle.
  - **Thm**: All radii of a circle are congruent.
- A line segment connecting two points on a circle is called a **chord**.
- A chord that passes through the center of a circle is called a **diameter**.
  - **Thm**: In a circle, the length of a diameter is twice that of a radius.

Example

- One vertex of ΔAMY is the center of the circle
  1. What is the name of the circle?
  2. How many vertices of the triangle are on the circle?
  3. What are AM and AY called with respect to the circle?
  4. What is MY called with respect to the circle?
  5. What kind of triangle must ΔAMY be?



More Circle Definitions

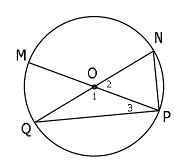
- **Congruent circles** are two or more circles that have congruent.
- **Concentric circles** are circles that have a common center.
- A continuous portion of a circle is called an **arc**.
- A **semicircle** is an arc whose endpoints are endpoints of a diameter the circle.
- An arc that is greater than a semicircle is called a **major arc**. An arc that is less than a semicircle is called a **minor arc**.
  - Minor arcs – denoted with 2 letters
  - Semicircles/Major arcs – denoted with 3 letters

Even More Circle Definitions

- An angle whose vertex is the center of a circle and whose sides are radii of the circle is called a **central angle**.
- The **intercepted arc** of an angle is determined by the two points of intersection of the angle with the circle and all points of the arc in the interior of the angle.

Example

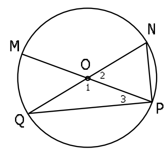
- MP and NQ intersect at O, the center of the circle. Name:
  1. 4 Radii
  2. 2 Diameters
  3. 4 Chords
  4. 1 Central Angle
  5. 1 Minor Arc
  6. 1 Semicircle
  7. 1 Major Arc
  8. Intercepted Arc of ∠MON
  9. Central angle that intercepts $\overset{\frown}{NP}$



Example

- QN is a diameter of ⊙O
- PN = ON = 12.

1. What kind of triangle is ΔQNP?
2. Find QP.



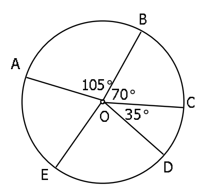
Measure of Arcs

- **Central Angle Postulate**: In a circle, the degree measure of a central angle is equal to the degree measure of its intercepted arc.
- The **measure of:**
  - A **minor arc** is the measure of its central angle
  - A **semicircle** is 180
  - A **major arc** is 360 minus the measure of the corresponding minor arc
  - A **circle** is 360
- If two arcs are in the same circle or congruent circles have the same measure, they are **congruent arcs**.
  - Why do we only talk about congruent arcs in the same circle?
- **Arc Addition Postulate**: If B lies between A and C on a circle, then $m\overset{\frown}{AB} + m\overset{\frown}{BC} = m\overset{\frown}{ABC}$.

Example

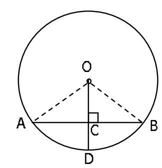
- In ⊙O, OE bisects ∠AOD. Find:

1. $m\overset{\frown}{AB}$
2. $m\overset{\frown}{BC}$
3. $m\overset{\frown}{BD}$
4. $m\angle\overset{\frown}{AOD}$
5. $m\overset{\frown}{AE}$
6. mACE
7. Whether $\overset{\frown}{AE} \cong \overset{\frown}{ED}$



Thm 6.1.1: A radius that is perpendicular to a chord bisects the chord.

- Given: OD ⊥ AB in ⊙O
- Prove: OD bisects AB

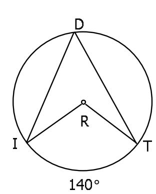


- Steps:
  - Construct OA and OB
  - ΔOAC ≃ ΔOBC
  - Conclusion?
- HW: Why is $\overset{\frown}{AD} \cong \overset{\frown}{DB}$?

Inscribed Angles

- An **inscribed angle** is an angle whose vertex is on a circle and whose sides are chords.
- Example:

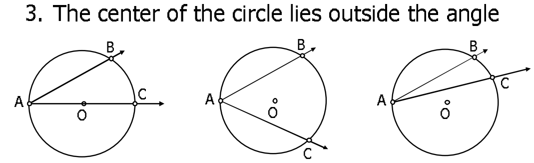
a. What is angle R called with respect to the circle?
b. What is the measure of angle R?
c. What is angle D called with respect to the circle?
d. What arc does angle D intercept?
e. In what arc is angle D inscribed?



Thm 6.1.2: The measure of an inscribed angle is ½ the measure of its intercepted arc.

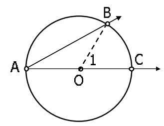
- Three Cases To Consider:

1. The center of the circle lies on a side of the angle
2. The center of the circle lies inside the angle
3. The center of the circle lies outside the angle



Proving Case 1 Theorem 6.1.2:

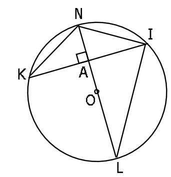
- Given: ∠BAC is an inscribed angle with $\overrightarrow{AC}$ passing through O, the center of the circle
- Prove: m∠A = ½ m$\overset{\frown}{BC}$
- Steps:
  - Draw $\overline{OB}$
  - Why is ΔAOB isosceles?
  - How does ∠1 relate to ∠A and ∠B?
  - How does m$\overset{\frown}{BC}$ relate to ∠1?



- HW: Prove Cases 2 and 3 using the result from Case 1 and the Arc Addition Postulate

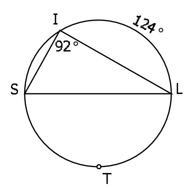
Example

- Assume $\overline{NL} \perp \overline{KI}$ in ⊙O
  a. Why does ∠K = ∠L?
  b. Why is ∠NIL a right angle? (*Hint: What is its intercepted arc?)*
  c. Why does KA = AI?
  d. Why is ΔNAK ≃ ΔNAI?
  e. Why does NK = NI?
  f. Why is m$\overset{\frown}{NK}$ = m$\overset{\frown}{NI}$?



Example

a. Is $\overset{\frown}{SIL}$ a semicircle?
b. What is m∠S?
c. Find m$\overset{\frown}{STL}$.
d. Find m∠L.
e. Find m$\overset{\frown}{IS}$.



Circle Theorems

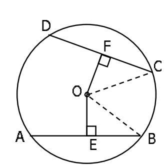
- **Thm 6.1.3**: In a circle, congruent minor arcs have congruent central angles.
- **Thm 6.1.4**: In a circle, congruent central angles have congruent arcs.
  - Draw radii to prove
- **Thm 6.1.5**: In a circle, congruent chords have congruent minor (major) arcs.
- **Thm 6.1.6**: In a circle, congruent arcs have congruent chords.

Inscribed Angle Theorems

- **Thm 6.1.9**: An angle inscribed in a semicircle is a right angle. (HW)
- **Thm 6.1.10**: If two inscribed angles intercept the same arc, then these angles are congruent. (HW)

Chord Distance Theorems

- **Distance** means length of perpendicular segment
- **Thm 6.1.7**: Chords that are the same distance from the center of the circle are congruent.
  - Prove by HL



- **Thm 6.1.8**: Congruent chords are located at the same distance from the center of a circle. (HW).
  - Prove triangles congruent by HL
    - H: radii are congruent
    - L: EB = FC since OE bisects AB and OF bisects DC

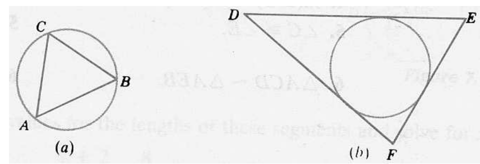
Lines In Circles

- A **tangent** is a line that intersects a circle at exactly one point (called the **point of tangency**).
- **Thm 6.2.3**: A line from the center of a circle drawn to a point of tangency is perpendicular to the tangent at that point. (Proved in book)
- A **secant** is a line that intersects a circle at exactly two points.

Inscribed Polygons

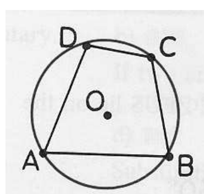
a. If a polygon's vertices are points on a circle and its sides are chords of the circle, the polygon is said to be **inscribed** within the circle.
   - Circle is **circumscribed** about the polygon

b. If all sides of the polygon are tangent to a circle, the polygon is said to be **circumscribed** about the circle.
   - Circle is **inscribed** in the polygon



Inscribed Quadrilaterals

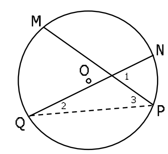
- **Theorem 6.2.1**:
  The opposite angles of an inscribed quadrilateral are supplementary.
  - **Given**: ABCD is an inscribed quadrilateral
  - **Prove**: ∠A is supplementary to ∠C
    - m∠A = ½ ?; m∠C = ½ ?
    - How do the two relate?



- **Corollary**: If a parallelogram is inscribed within a circle, then it is a rectangle.

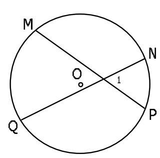
Angles Formed By Chords

- **Thm 6.2.2**: The measure of an angle formed by two chords that intersect within a circle is ½ the sum of the measures of the arcs intercepted by the angle and its vertical angle.
  - Ie: m∠1 = ½ ($m\overset{\frown}{PM}$ + $m\overset{\frown}{QN}$)
  - m∠1 = m∠2 + m∠3
  - m∠2 = ½ $m\overset{\frown}{NP}$
  - m∠3 = ½ $m\overset{\frown}{QN}$



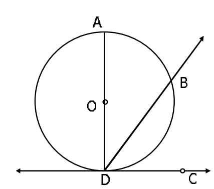
Example

- Find m∠1 if $m\overset{\frown}{NP}$ = 80° and $m\overset{\frown}{MQ}$ = 110°



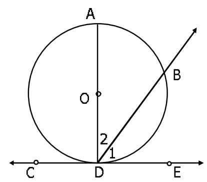
Angles Formed by Chords and Tangents

- **Cor 6.2.4**: The measure of an angle formed by a tangent and a chord drawn to the point of tangency is ½ the measure of the intercepted arc.
  - m∠BDC = ½ $m\overset{\frown}{DB}$



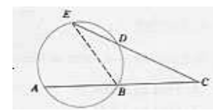
Example

- ⊙O with diameter $\overline{AD}$ and $m\widehat{AB} = 84°$
- Find:
  1. m∠2
  2. m∠1
  3. m∠ADC
  4. m∠CDB



Angles Formed By Secants

- **Thm 6.2.5**: The measure of an angle formed when two secants intersect at a point *outside* the circle is ½ the difference of the measures of the two intercepted arcs.
  - Use Measure of Inscribed Angles
  - sum(angles of triangle) = sum(angles on a straight line)

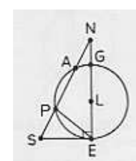


Theorems for Angles on Exterior of Circle

- **Thm 6.2.6**: Angle formed by secant and tangent intersecting in exterior of circle…
  - Angle = ½ (difference of intercepted arcs)
- **Thm 6.2.7**: Angle formed by two intersecting tangents…
  - Angle = ½ (difference of intercepted arcs)

Example

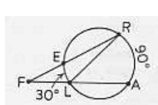
- SE tangent to ⊙L
- $m\widehat{AP} = m\widehat{PE}$



a. Why is ∠SEN a right angle?
b. Why is m∠N = ½($m\widehat{PE} - m\widehat{AG}$)?
c. Why is AP = PE?
d. Why is m∠PEG = ½ $m\widehat{PG}$?
e. Why is $m\widehat{PA} + m\widehat{AG} = m\widehat{PG}$?

Example

a. Find m∠R
b. Find m∠F
c. Find m∠RLA
d. Why should m∠RLA = m∠R + m∠F?



Homework

- Due Monday 7/12
  - Read Sections 6.1 and 6.2
  - 6.1: #1-11, 13, 18-21, 28-30, 31 (hint: Use AA~), 32-35, 35, 37, 39, 40
  - 6.2: #1-5, 8, 9, 12-16
- Exam 4 – Monday, July 12
  - Covering Chapters 5; Sections 6.1, 6.2
- Suggested Preparation:
  - Do HW above
  - Chapter 5 Review: #2-4, 6-8, 13-15, 16-26, 29-39, 41