Math 119 – Plane Geometry

Sections 6.2 and 6.3
Circles II
7/12/2004

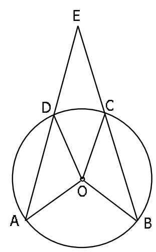
Recap For Measuring Angles

Location of Vertex	**Measure of Angle**
Center of circle	Measure of Intercepted Arc
Interior of Circle	½ Sum of Intercepted Arcs
Exterior of Circle	½ Difference of Intercepted Arcs
On Circle	½ Intercepted Arc

Example

- ⊙O
- m∠AOB = 136°
- m∠DOC = 46°

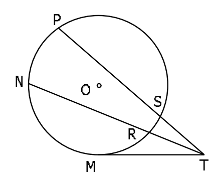
- Find: m∠E



Example

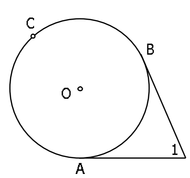
- m$\overset{\frown}{MN}$ = 70°
- m$\overset{\frown}{NP}$ = 88°
- m$\overset{\frown}{MR}$ = 46°
- m$\overset{\frown}{RS}$ = 26°

- Find:
  1. m∠MTN
  2. m∠NTP
  3. m∠MTP



Example

- m∠1 = 46°

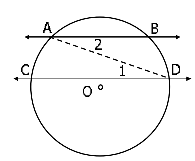


- Find m$\overset{\frown}{AB}$ and m$\overset{\frown}{ABC}$
  - Hint: Consider m$\overset{\frown}{AB}$ + m$\overset{\frown}{ABC}$

- **Q**: What if $\overline{AB}$ formed a diameter? What would be the measure of ∠1?

Thm 6.2.8: If two parallel lines intersect a circle, the intercepted arcs between these lines are congruent. (HW)

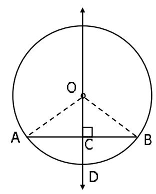
- Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
- Prove: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$



- Helpful Hints:
  - Why are angles 1 and 2 congruent?
  - Express each in terms of the intercepted arc.

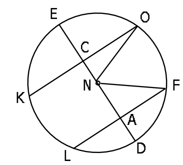
Perpendicular Lines Through Center

- **Recall**: A radius that is perpendicular to a chord bisects the chord.
- **Thm 6.3.1**: If a line is drawn through the center of a circle perpendicular to a chord, then it bisects the chord and its arc. (HW)
- Given: OD $\perp$ AB in $\odot$O
- Prove: $\overline{CA} \cong \overline{CB}$ and $\overset{\frown}{CA} \cong \overset{\frown}{CB}$
- Use: HL and cpctc



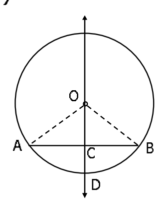
Example

- In $\odot$N, $\overline{DE} \perp \overline{FL}$ and $\overline{DE} \perp \overline{OK}$; DE = 50, FL = 30, and OK = 48
  1. Why does $\overline{DE}$ bisect $\overline{FL}$ and $\overline{OK}$?
  2. Find the following:
     a. NF
     b. NA
     c. NC
  3. If two chords have different lengths, which chord is closer to the center?



Line Through Center Bisect Chord

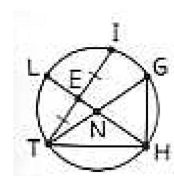
- **Thm 6.3.2**: If a line through the center of a circle bisects a chord other than a diameter, then it is perpendicular to the chord. (HW)
- Given: $\overleftrightarrow{OD}$ bisects $\overline{AB}$ in $\odot$O
- Prove: $\overline{AB} \perp \overleftrightarrow{OD}$
- Use: SSS and cpctc



- **Q**: Why can't the chord be a diameter?

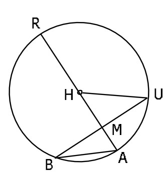
Example

a. Why is LH $\perp$ TI?
b. Why does m$\overset{\frown}{GH}$ = m$\angle$GNH?
c. Why is LN = NG?
d. Why is $\angle$THG a right angle?



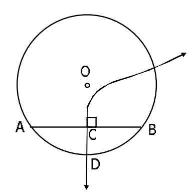
Example

- In $\odot$H, RA bisects BU; HU = 5 and HM = 3
  1. Why is RA $\perp$ BU?
  2. Find the following
     a. RA
     b. BU
     c. BA
     d. MU
     e. MA



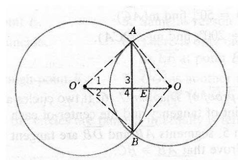
Thm 6.3.3: The perpendicular bisector of a chord contains the center of the circle.

- Given: $\overleftrightarrow{DC}$ is the perpendicular bisector of $\overline{AB}$ in $\odot$O
- Prove: $\overleftrightarrow{DC}$ contains point O
- Indirect Proof
  - Assume: $\overleftrightarrow{DC}$ does not contain O
  - Draw $\overleftrightarrow{OC}$
  - Why is $\overleftrightarrow{OC} \perp \overline{AB}$?
  - Why is this a contradiction?
  - What does this tell us about the assumption?



Example

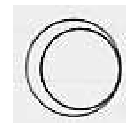
- Circles O and O′ intersect at A and B.
  1. Is $\overline{OO'}$ the perpendicular bisector of $\overline{AB}$?
  2. Is $\overline{AB}$ the perpendicular bisector of $\overline{OO'}$?



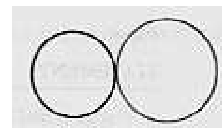
Circles That Are Tangent

a. If two circles touch at one point, they are **tangent circles**.

b. If one circle is on the interior of the other, except for a point of tangency, the circles are **tangent internally**.

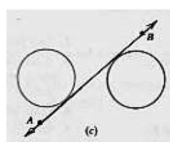


c. Otherwise, they are **tangent externally**.

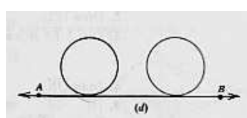


Common Tangents

a. A tangent is a **common internal tangent** if it is tangent to 2 circles on opposite sides of it.

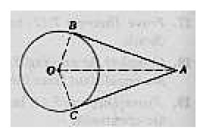


b. A tangent is a **common external tangent** if it is tangent to 2 circles on the same side of it.



Thm 6.3.4: The tangent segments to a circle from an external point are congruent.

- Given: $\overline{AB}$ and $\overline{AC}$ are tangents to circle O
- Prove: AB = AC

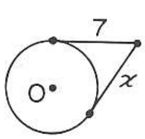


- Steps:
  - Draw radii $\overline{OB}$ and $\overline{OC}$. Also draw segment $\overline{OA}$.
  - Use HL and cpctc

Example

- Tangent segments have been drawn to the circles in the figures. Find x in each of the following:

1.



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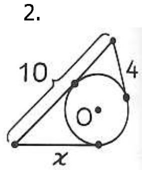
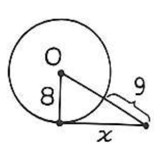


3.



Example

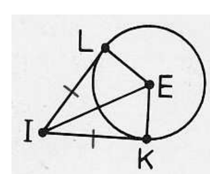
- $\overline{IK}$ is tangent to circle E and IL = IK

a. Why is ΔILE ≃ ΔIKE?

b. Why is ∠K a right angle?

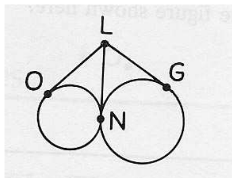
c. Why is ∠L a right angle?

d. Why is $\overline{IL}$ tangent to ⊙E?



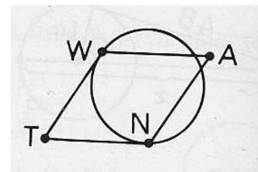
Example: A Proof

- **Given**: $\overline{LO}$, $\overline{LN}$, and $\overline{LG}$ are tangent segments to the circles
- **Prove**: $\overline{LO} \cong \overline{LG}$



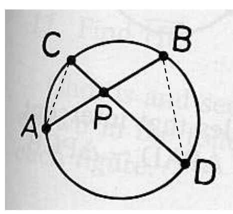
Example: Another Proof

- **Given**: $\overline{TW}$ and $\overline{TN}$ are tangent to the circle; WANT is a parallelogram
- **Prove**: WANT is a rhombus



The Intersecting Chords Theorem

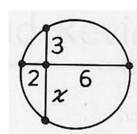
- **Theorem 6.3.5**: If two chords intersect within a circle, then the product of the lengths of the segments (parts) of one chord is equal to the product of the lengths of the segments of the other chord.
- **Given**: Chords $\overline{AB}$ and $\overline{CD}$ intersect at point P in the circle
- **Prove**: PA * PB = PC * PD
- **Note**:
  - $m\angle A \cong m\angle C$ and $m\angle D \cong m\angle B$
  - Why?
  - What arc does each intercept?
- **Q**: What does having two angles congruent get us?
- Apply CSSTP.



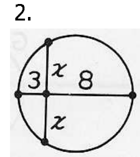
Example

- Chords have been drawn in the following figures. Find x.

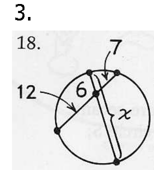
1.



2.



3.



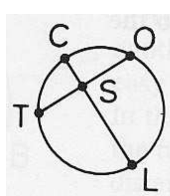
Example

- Chords $\overline{CL}$ and $\overline{TO}$ intersect at point S in the circle

a. Write an equation for this figure that follows from the Intersecting Chords Theorem.

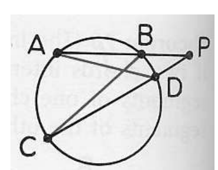
b. Use one of the theorems from last time to write a formula for $m\angle OSL$.

c. If $\overline{CL}$ is a diameter of the circle and bisects $\overline{TO}$, what can you conclude?



The Secant Segments Theorem

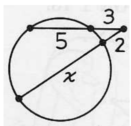
- **Theorem 6.3.6**: If two secants are drawn to a circle from an external point, then the products of the lengths of each secant with its external segment are equal.
- **Given**: The circle with secants $\overline{PA}$ and $\overline{PC}$
- **Prove**: PA * PB = PC * PD
- **Draw**: $\overline{CB}$ and $\overline{AD}$
- **Note**: $\angle A \cong \angle C$. Why?
- **Q**: Why can we now conclude $\Delta PAD \sim \Delta PCB$?
- Use CSSTP



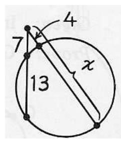
Example

- ►Secants have been drawn in the following figures. Find x.

1.



2.



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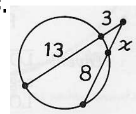
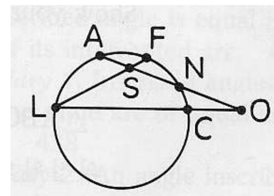


Example

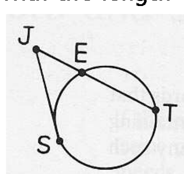
- ► $\overline{AO}$ and $\overline{LO}$ are secant segments to the circle and $\overline{LF}$ is a chord.
- ► FS = 4, SL = 15, LC = 24, CO = 9, and NO = 11.

a. Find AO
b. Find AN
c. Find AS



Related Theorem in Your Book

- ►We won't prove this in class, but…
- ►**<u>Theorem 6.3.7</u>**: If a secant and a tangent are drawn to a circle, then the square of the length of the tangent equals the product of the length of the secant with the length of its external segment.



Homework

- ►Due Tuesday 7/13
  - Read Sections 6.2 and 6.3
  - 6.2: #6, 7, 10, 11, 17-22, 31, 34, 35, 39
  - 6.3: #1-5, 7-25, 38, 39, 40