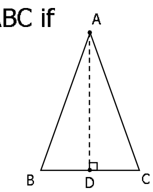
Math 119 – Plane Geometry

Sections 7.2 and 7.3
Area II
7/15/2004

Recall: Perimeter

- The **perimeter** of a polygon is the sum of the lengths of all the sides of the polygon.
- **Ex**: Find the perimeter of ΔABC if
  1. AB = 5, AC = 6, and BC = 7
  2. Altitude AD = 12, BC = 10 and AB = AC



Area of a Triangle: Heron's Formula

- **Heron's Formula**: If the three sides of a triangle have lengths a, b, and c, then the area of the triangle is given by
  $$A = \sqrt{s(s-a)(s-b)(s-c)}$$
  where the semiperimeter of the triangle is
  $$s = \frac{1}{2}(a + b + c).$$
- **Ex**: Find the area of a triangle which has sides of lengths 4, 13, and 15.
- **Ex**: The sides of the Bermuda Triangle are 1,000, 1,000 and 1,100 miles in length. Find the area of the Bermuda Triangle.

Example

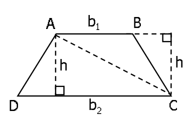
- Suppose that a triangle has lengths 4, 6, and 10.
  1. Use Heron's Formula to find its area.
  2. Why does the result turn out as it does?

Area of a Trapezoid

- **Thm 7.2.2**: The area A of a trapezoid whose bases have lengths b_1 and b_2 and whose altitude has length h is given by

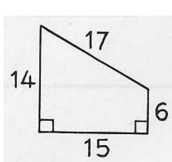
$$A = \frac{1}{2} h(b_1 + b_2).$$

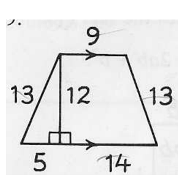
  - Calculate A_{ABC} and A_{ADC}
  - Use Area Addition Postulate



Example

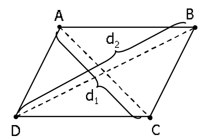
- Find the area of each figure:





Area of a Rhombus

- **<u>Thm 7.2.4</u>**: The area A of a rhombus whose diagonals have lengths d_1 and d_2 is given by $A = \frac{1}{2} d_1 d_2$.
  - Calculate A_{ABC} and A_{ACD}
  - Use Area Addition Postulate



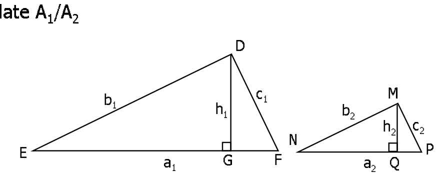
- **<u>Ex</u>**: Find A_{ABCD} if AC = 12 and BD = 16

Example

- Find the areas of the following figures:
  1. A square whose perimeter is 100.
  2. A rectangle whose base is 5 and whose perimeter is 16.
  3. A rhombus whose diagonals are 8 and 9.
  4. A triangle whose sides are 10, 17, and 21.
  5. A right triangle whose hypotenuse is 41 and one of whose legs is 9.

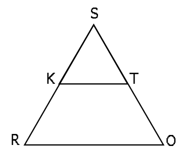
Areas of Similar Polygons

- **<u>Thm 7.2.6</u>**: The ratio of the areas of two similar triangles equals the square of the ratio of the lengths of any two corresponding sides; that is, $A_1/A_2 = (a_1/a_2)^2$.
  - **<u>Recall</u>**: Altitudes of similar triangles have the same ratio as any pair of corresponding sides.
  - Calculate A_1/A_2



Example

- ΔSOR is equilateral, with midsegment KT



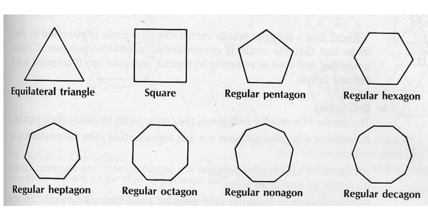
1. Find A_{STK}
2. Find A_{SOR}
3. Find A_{OTKR}
4. How does the area of ΔSTK compare with the area of ΔSOR?
5. How does the area of ΔSTK compare with the area of OTKR?

Example

- Use the ratio A_1/A_2 to compare the areas of
  1. Two similar triangles in which the sides of the first triangle are ½ as long as the sides of the second triangle.
  2. Two squares in which each side of the first square is 3 times as long as each side of the second square.

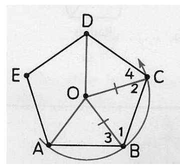
Recall: Regular Polygons

- A **<u>regular polygon</u>** is a polygon that is both equilateral and equiangular.



Circumscribing Circles About Polygons

- **Thm 7.3.1(a)**: A circle can be circumscribed about any regular polygon.
  - Why can we draw a circle containing A, B, and C?
  - Why is ∠1 ≅ ∠2?
  - Why is ∠ABC ≅ ∠BCD?
  - Why is ∠3 ≅ ∠4?
  - Why is $\overline{AB} \cong \overline{CD}$?
  - Why is ΔOBA ≅ ΔOCD?
  - Why is $\overline{OD} \cong \overline{OA}$?
  - Why is $\overline{OD}$ a radius?
  - Why does D lie on circle O?
  - How can we prove E is on the circle?

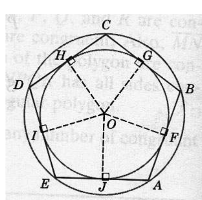


Example 5: True or False

a. A circle can be circumscribed about every triangle.
b. If a triangle is equilateral, it must be regular.
c. If a quadrilateral is equilateral, it must be regular.
d. If a quadrilateral is equiangular, it must be regular.
e. If a quadrilateral is equiangular, then a circle can be circumscribed about it.
f. If a polygon is regular, a circle can be circumscribed about it.
g. If a circle can be circumscribed about it a polygon, it must be regular.
h. If a polygon is regular, it must be equilateral.
i. If a polygon is equilateral, it must be equiangular.

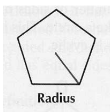
Inscribing Circles in Regular Polygons

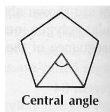
- **Thm 7.3.1(b)**: A circle can be inscribed in any regular polygon.
  - Construct circle O by last theorem
  - Construct perpendiculars from O to $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{EA}$
  - Why is $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EA}$?
  - Why is $\overline{OF} \cong \overline{OG} \cong \overline{OH} \cong \overline{OI} \cong \overline{OJ}$?
  - Why does this make the smaller circle?
  - Why are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{EA}$ tangents to the smaller circle?
  - Why is the smaller circle inscribed in the polygon?

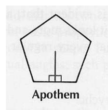


Vocabulary

- The **center of a regular polygon** is the common center for the inscribed and circumscribed circles of the polygon.
- A **radius of a regular polygon** is any line segment that joins the center of the polygon to one of its vertices.
- A **central angle of a regular polygon** is an angle formed by two consecutive radii of the polygon.
- An **apothem of a regular polygon** is any line segment drawn from the center of the polygon perpendicular to one of the sides.

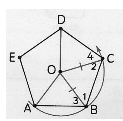

Radius


Central angle

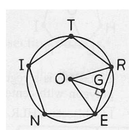

Apothem

Theorems

- **Thm 7.3.3**: Any radius of a regular polygon bisects the angle at the vertex to which it is drawn.

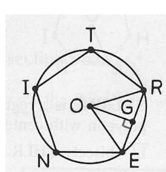


- **Thm 7.3.4**: Any apothem of a regular polygon bisects the side of the polygon to which it is drawn.



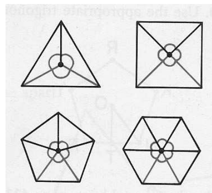
Example

- Circle O is circumscribed about regular pentagon NITRE; $\overline{OG} \perp \overline{RE}$
  a. What is point O called with respect to the pentagon?
  b. What is $\overline{OG}$ called?
  c. How do we know $\overline{OG}$ bisects $\overline{RE}$?
  d. What are $\overline{OR}$ and $\overline{OE}$ called with respect to NITRE?
  e. What kind of triangle is ΔORE?
  f. What is $\angle ROE$ called with respect to NITRE?
  g. Does $\overline{OG}$ bisect $\angle ROE$?



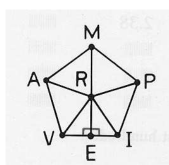
Measures of Central Angles of Regular Polygons

- As the number of sides of a regular polygon increases, how does the measure of one of its central angles change?
- Find the measure of a central angle of each figure to the right.
- Find the measure of a central angle of a regular decagon.
- How would you express the measure of a central angle of a regular polygon that has n sides in terms of n?
- This gives Theorem 7.3.2…



Area of a Regular Polygon

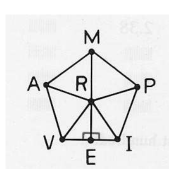
- **Thm 7.3.5**: The area A of a regular polygon whose apothem has length a and whose perimeter is P is given by A = ½ aP.
  - Cut the polygon into right triangles using the apothems
  - Add up the areas



Example

- VAMPI is a regular pentagon with center R; $\overline{RE} \perp \overline{VI}$.

a. What is $\overline{RE}$ called with respect to VAMPI?
b. What is $\overline{RE}$ called with respect to triangle VRI?
c. Why are triangles VRI, IRP, PRM, MRA, and ARV congruent?
d. Why are the areas of those triangles equal?
e. How does A_{VAMPI} compare to A_{VRI}?
f. Write A_{VRI} in terms of $\overline{VI}$ and $\overline{RE}$.
g. Write A_{VAMPI} in terms of $\overline{VI}$ and $\overline{RE}$.



Example

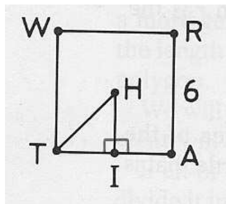
► WRAT is a square with center H; RA = 6

a. Use the formula for area of a square to find A_{WRAT}.

b. Find TI.

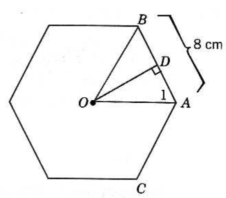
c. Find HT.

d. Use the new theorem to find A_{WRAT}.



Example

► A side of the regular hexagon is 8 cm long. Find its area.



Homework

► Due Monday 7/19
- Read Sections 7.2 and 7.3
- 7.2: #1-14, 17-28, 41
- 7.3: #3-26

► Exam 5 – Monday, July 19
- Covering Chapters 6; Sections 7.1 – 7.3

► Suggested Preparation:
- Do HW above
- Chapter 6 Review: #1-35, 36(a), 37-51