

Math 119 – Plane Geometry

Sections 7.2 and 7.3
Area II
7/15/2004

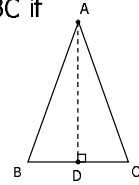
Recall: Perimeter

► The **perimeter** of a polygon is the sum of the lengths of all the sides of the polygon.

► **Ex:** Find the perimeter of $\triangle ABC$ if

1. $AB = 5$, $AC = 6$, and $BC = 7$

2. Altitude $AD = 12$, $BC = 10$
and $AB = AC$



Area of a Triangle: Heron's Formula

► **Heron's Formula:** If the three sides of a triangle have lengths a , b , and c , then the area of the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where the semiperimeter of the triangle is
 $s = \frac{1}{2}(a + b + c)$.

► **Ex:** Find the area of a triangle which has sides of lengths 4, 13, and 15.

► **Ex:** The sides of the Bermuda Triangle are 1,000, 1,000 and 1,100 miles in length. Find the area of the Bermuda Triangle.

Example

► Suppose that a triangle has lengths 4, 6, and 10.

1. Use Heron's Formula to find its area.

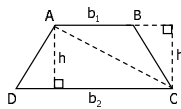
2. Why does the result turn out as it does?

Area of a Trapezoid

► **Thm 7.2.2:** The area A of a trapezoid whose bases have lengths b_1 and b_2 and whose altitude has length h is given by

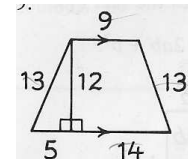
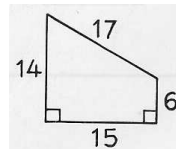
$$A = \frac{1}{2} h(b_1 + b_2).$$

- Calculate A_{ABC} and A_{ADC}
- Use Area Addition Postulate



Example

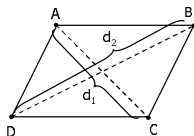
► Find the area of each figure:



Area of a Rhombus

► **Thm 7.2.4:** The area A of a rhombus whose diagonals have lengths d_1 and d_2 is given by $A = \frac{1}{2} d_1 d_2$.

- Calculate A_{ABC} and A_{ACD}
- Use Area Addition Postulate



► **Ex:** Find A_{ABCD} if $AC = 12$ and $BD = 16$

Example

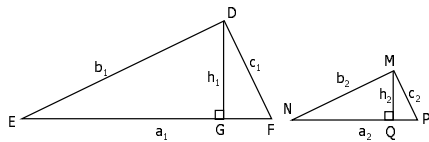
► Find the areas of the following figures:

1. A square whose perimeter is 100.
2. A rectangle whose base is 5 and whose perimeter is 16.
3. A rhombus whose diagonals are 8 and 9.
4. A triangle whose sides are 10, 17, and 21.
5. A right triangle whose hypotenuse is 41 and one of whose legs is 9.

Areas of Similar Polygons

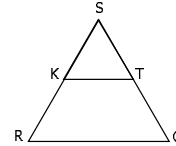
► **Thm 7.2.6:** The ratio of the areas of two similar triangles equals the square of the ratio of the lengths of any two corresponding sides; that is, $A_1/A_2 = (a_1/a_2)^2$.

- **Recall:** Altitudes of similar triangles have the same ratio as any pair of corresponding sides.
- Calculate A_1/A_2



Example

► Δ SOR is equilateral, with midsegment KT



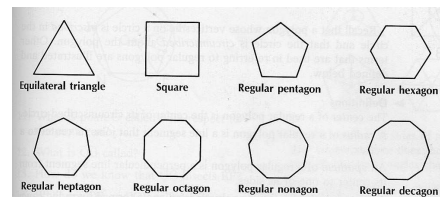
1. Find $A_{\Delta STK}$
2. Find $A_{\Delta SOR}$
3. Find $A_{\Delta OTKR}$
4. How does the area of Δ STK compare with the area of Δ SOR?
5. How does the area of Δ STK compare with the area of Δ OTKR?

Example

- Use the ratio A_1/A_2 to compare the areas of
1. Two similar triangles in which the sides of the first triangle are $\frac{1}{2}$ as long as the sides of the second triangle.
 2. Two squares in which each side of the first square is 3 times as long as each side of the second square.

Recall: Regular Polygons

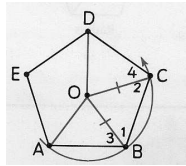
► A **regular polygon** is a polygon that is both equilateral and equiangular.



Circumscribing Circles About Polygons

► **Thm 7.3.1(a):** A circle can be circumscribed about any regular polygon.

- Why can we draw a circle containing A, B, and C?
- Why is $\angle 1 \cong \angle 2$?
- Why is $\triangle ABC \cong \triangle BCD$?
- Why is $\angle 3 \cong \angle 4$?
- Why is $\overline{AB} \cong \overline{CD}$?
- Why is $\triangle OBA \cong \triangle OCD$?
- Why is $\overline{OD} \cong \overline{OA}$?
- Why is \overline{OD} a radius?
- Why does D lie on circle O?
- How can we prove E is on the circle?



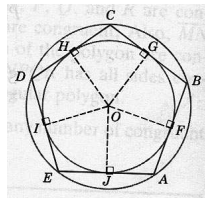
Example 5: True or False

- A circle can be circumscribed about every triangle.
- If a triangle is equilateral, it must be regular.
- If a quadrilateral is equilateral, it must be regular.
- If a quadrilateral is equiangular, it must be regular.
- If a quadrilateral is equiangular, then a circle can be circumscribed about it.
- If a polygon is regular, a circle can be circumscribed about it.
- If a circle can be circumscribed about a polygon, it must be regular.
- If a polygon is regular, it must be equilateral.
- If a polygon is equilateral, it must be equiangular.

Inscribing Circles in Regular Polygons

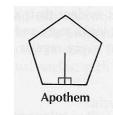
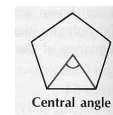
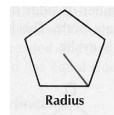
► **Thm 7.3.1(b):** A circle can be inscribed in any regular polygon.

- Construct circle O by last theorem
- Construct perpendiculars from O to \overline{AB} , \overline{BC} , \overline{CD} , \overline{DE} , and \overline{EA}
- Why is $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EA}$?
- Why is $\overline{OF} \cong \overline{OG} \cong \overline{OH} \cong \overline{OI} \cong \overline{OJ}$?
- Why does this make the smaller circle?
- Why are \overline{AB} , \overline{BC} , \overline{CD} , \overline{DE} , and \overline{EA} tangents to the smaller circle?
- Why is the smaller circle inscribed in the polygon?



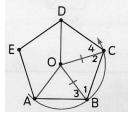
Vocabulary

- The **center of a regular polygon** is the common center for the inscribed and circumscribed circles of the polygon.
- A **radius of a regular polygon** is any line segment that joins the center of the polygon to one of its vertices.
- A **central angle of a regular polygon** is an angle formed by two consecutive radii of the polygon.
- An **apothem of a regular polygon** is any line segment drawn from the center of the polygon perpendicular to one of the sides.

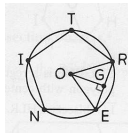


Theorems

- ▶ **Thm 7.3.3:** Any radius of a regular polygon bisects the angle at the vertex to which it is drawn.



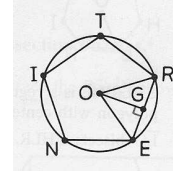
- ▶ **Thm 7.3.4:** Any apothem of a regular polygon bisects the side of the polygon to which it is drawn.



Example

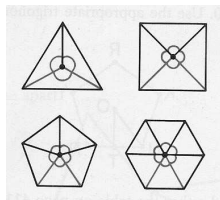
- ▶ Circle O is circumscribed about regular pentagon NITRE; $\overline{OG} \perp \overline{RE}$

- What is point O called with respect to the pentagon?
- What is \overline{OG} called?
- How do we know \overline{OG} bisects \overline{RE} ?
- What are \overline{OR} and \overline{OE} called with respect to NITRE?
- What kind of triangle is $\triangle ORE$?
- What is $\angle ROE$ called with respect to NITRE?
- Does \overline{OG} bisect $\angle ROE$?



Measures of Central Angles of Regular Polygons

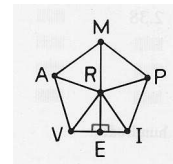
- ▶ As the number of sides of a regular polygon increases, how does the measure of one of its central angles change?
- ▶ Find the measure of a central angle of each figure to the right.
- ▶ Find the measure of a central angle of a regular decagon.
- ▶ How would you express the measure of a central angle of a regular polygon that has n sides in terms of n?
- ▶ This gives Theorem 7.3.2...



Area of a Regular Polygon

- ▶ **Thm 7.3.5:** The area A of a regular polygon whose apothem has length a and whose perimeter is P is given by $A = \frac{1}{2} aP$.

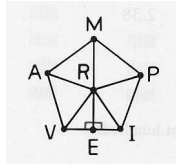
- Cut the polygon into right triangles using the apothems
- Add up the areas



Example

► $VAMP$ is a regular pentagon with center R ; $\overline{RE} \perp \overline{VI}$.

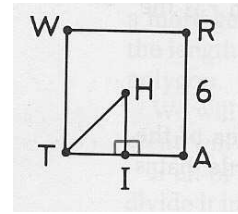
- What is \overline{RE} called with respect to $VAMP$?
- What is \overline{RE} called with respect to triangle VRI ?
- Why are triangles VRI , IRP , PRM , MRA , and ARV congruent?
- Why are the areas of those triangles equal?
- How does A_{VAMP} compare to A_{VRI} ?
- Write A_{VRI} in terms of \overline{VI} and \overline{RE} .
- Write A_{VAMP} in terms of \overline{VI} and \overline{RE} .



Example

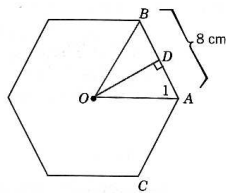
► $WRAT$ is a square with center H ; $RA = 6$

- Use the formula for area of a square to find A_{WRAT} .
- Find TI .
- Find HT .
- Use the new theorem to find A_{WRAT} .



Example

► A side of the regular hexagon is 8 cm long. Find its area.



Homework

- Due Monday 7/19
 - Read Sections 7.2 and 7.3
 - 7.2: #1-14, 17-28, 41
 - 7.3: #3-26
- Exam 5 – Monday, July 19
 - Covering Chapters 6; Sections 7.1 – 7.3
- Suggested Preparation:
 - Do HW above
 - Chapter 6 Review: #1-35, 36(a), 37-51