Math 119 – Plane Geometry

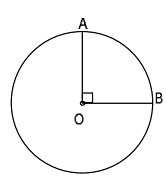
Sections 7.4 and 7.5
Area III
7/20/2004

Circumference

- **Post 22**: The ratio of the circumference of a circle to the length of its diameter is a unique positive constant.
  - Define π to be this ratio: $\pi = C/d$ in any circle
  - π is irrational
    - Approximations: 22/7, 3.14
- **Thm 7.4.1**: The circumference of a circle is given by the formula $C = \pi d$ or $C = 2\pi r$.

Example

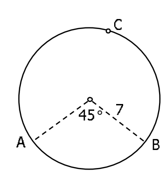
- In ⊙O, OA = 7. Using $\pi \approx 22/7$
  a) Find the approximate circumference C of ⊙O
  b) Find the approximate length of the minor arc AB



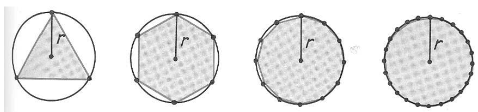
- The exact circumference of a circle is 17π.
  a) Find the length of the radius
  b) Find the length of the diameter

Length of An Arc

- **Thm 7.4.2**: In a circle whose circumference is C, the length ℓ of an arc whose degree measure is m is given by $\ell = m/360 * C$.
- **Ex**: Find the approximate length of major arc ABC in a circle of radius 7 if $m\widehat{AC} = 45°$. Use $\pi = 22/7$.



Area of a Circle

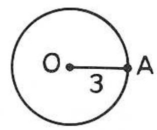


- As the number of sides of a regular polygon inscribed in a circle increases, the area of the polygon becomes a better and better approximation for the area of a circle.
  - Recall: The **circumference of a circle** is the distance around the circle. $C = 2\pi r$
  - A(regular polygon) = ½ Pr
  - A(circle) = ½ (C r) = πr^2

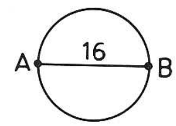
Example

a. A circle has an area of 64π. Find its diameter.

b. Find the area of each of the following:



$\overline{OA}$ is a radius.



$\overline{AB}$ is a diameter.

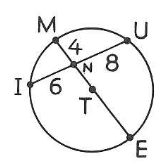
Example

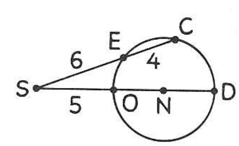
- As the number of sides of a regular polygon inscribed in a circle increases,
  - a. What measurement of the circle do the perimeters of the polygons approach?
  - b. What measurement of the circle do the areas of the polygons approach?

Example

- $\overline{IU}$ and $\overline{ME}$ are chords of circle T.
- $\overline{SC}$ and $\overline{SD}$ are secants to circle N.

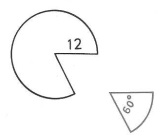
a. Find ME.
b. Find the area of circle T.
c. Find ND.
d. Find the area of circle N.





Sectors

- A **sector of a circle** is a region bounded by two radii of the circle and an arc intercepted by those radii.
  - (A slice of pizza)

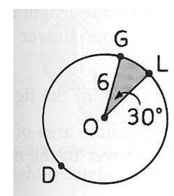


- **Post 23**: The ratio of the degree measure m of the central angle of a sector to 360° is the same as the ratio of the area of the sector to the area of the circle.
  - A(sector)/A(circle) = (central angle of sector)/360

Example

- In circle O, OG = 6 and m∠O = 30°.

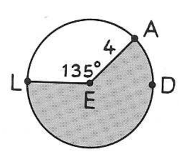
a. Find the circumference of the circle.
b. Find $m\overset{\frown}{GL}$.
c. Find $m\overset{\frown}{GDL}$.
d. Find the area of the circle.
e. Find the area of the shaded sector.



Example

- In circle E, EA = 4 and m∠E = 135°.

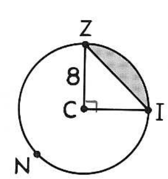
a. Find the circumference of the circle.
b. Find the $m\overset{\frown}{LDA}$.
c. Find the area of the circle.
d. Find the area of the shaded sector.



Segment Example

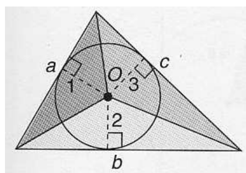
- A **segment of a circle** is the area enclosed by an arc and its respective chord.

a. Find the area of the circle.
b. Find the area of sector ZCI.
c. Find the area of ΔZCI.
d. Find the area of the shaded region (the segment).
e. Find the area of the region bounded by $\overline{ZI}$ and $\overset{\frown}{ZNI}$.



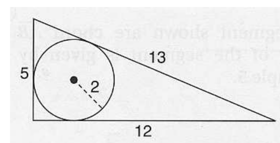
Area of A Triangle With an Inscribed Circle

- **Thm 7.5.3**: Where P represents the perimeter of a triangle and r represents the length of the radius of its inscribed circle, the area of the triangle is given by
  $A = \frac{1}{2} rP$.
  - Calculate area of each small triangle



Example

- Find the area of a triangle whose sides measure 5, 12 and 13 if the radius of the inscribed circle is 2.
  - Use the latest theorem
  - Use Heron's Formula
  - Use formula for area of triangle

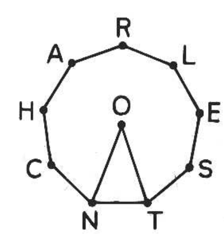


Review Example: True or False

a. An apothem of a regular polygon bisects one of its sides.
b. A circle can be circumscribed about a polygon if, and only if, the polygon is regular.
c. Each central angle of a regular polygon having n sides has a measure of 360/n.
d. The ratio of the circumference to the diameter of a circle does not depend on the circle's size.
e. The area of a circle is $1/4\pi d^2$, where d is the length of its diameter.

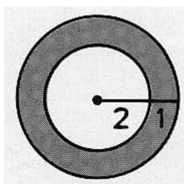
Example

- CHARLESTN is a regular nonagon with center O. Find the measure of each of the following angles:
  a. $\angle$O
  b. $\angle$ONT
  c. $\angle$CNT

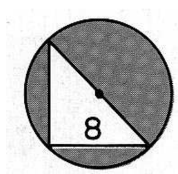


Example

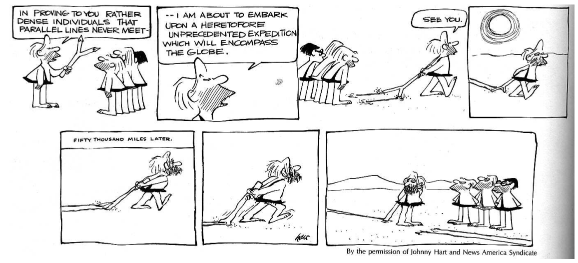
- ► Find the area of the shaded region in each figure.
  - a. The circles are concentric.



  - b. An isosceles right triangle is inscribed in the circle.



Things to Think About…



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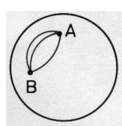
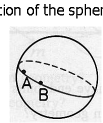
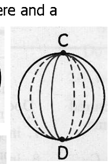
- ► Euclid defined parallel lines as "lines that, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction."

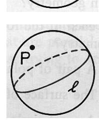
Great Circles



- ► Distance between two points
  - On plane: Measured along the line determined by the two points
  - On surface of sphere: **Q**: Which curved path to measure by? **A**: The shortest
    - ► A **great circle** is the locus of the intersection of the sphere and a plane containing the sphere's center.
    - ► Lines on the Sphere
    - ► Do 2 points determine a line on a sphere?
    - ► Through P, how many lines can be drawn parallel to l?
      - Parallel lines lie in same plane but do not intersect
      - Consider sphere to be plane
      - Parallel Postulate doesn't apply!







Homework

- ► Due Wednesday 7/21
  - Read Sections 7.4 and 7.5
  - 7.4: #1-12, 16, 21-31, 40
  - 7.5: #1-18, 21, 25